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**ANONYMITY, EQUAL TREATMENT, AND OVERCONFIDENCE:
CONSTRAINTS ON COMMUNICATION MAY ENHANCE
INFORMATION TRANSMISSION**

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Anonymity, Equal Treatment, and Overconfidence: Constraints on Communication May Enhance Information Transmission

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Abstract

This paper offers a simple but rich framework to study communication subject to various constraints such as anonymity requirements, equal treatment of multiple agents, overconfidence of an expert, and garbling, by extending the cheap talk model of Crawford and Sobel (1982). Common to these seemingly distinct types of constraints in communication is that the action by a decision maker is less sensitive to a message than without such constraints. Reduced sensitivity can alter the structure of informative equilibria dramatically, and leads to a type of informational distortion, termed incentives to *exaggerate*, which differs qualitatively from the well-known incentives to *overstate/understate*. We demonstrate that the two different types of distortion may partly offset each other, so the introduction of the constraints may be beneficial when the level of conflict between communicating parties is large. Our model can also be applied to study communication in public good provision where equal treatment is often implicitly assumed.

Keywords: Cheap Talk, Anonymity, Overconfidence, Equal Treatment, Public Good Provision, Noisy Communication, Exaggeration

JEL Classifications: D71, D82, D83

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1 Introduction

A great deal of information in society is communicated anonymously. In firms or schools, junior members often communicate with senior members (management, teachers) anonymously, through anonymous questionnaires, unions, representatives, or third parties such as external consultants so that the sender of a message may not be known to its receiver. Are we more likely, or less likely to tell the truth when we are anonymous than otherwise? Why is anonymous communication so widely used?

Firms or legislative bodies often hire consultants to obtain expert information, but apart from the possibility that they may be biased depending on their personal preference or political stance, psychological studies have found that experts tend to be "overconfident" about the information they have (e.g. Kahneman et al., 1982; Griffin and Tversky, 1992). How should a report from an overconfident expert be interpreted? Are bias and overconfidence qualitatively different? Is overconfidence always bad?

Suppose that a decision maker chooses the quality or quantity of a good that is equally consumed by all members of a group with different preferences. Before making her decision, the decision maker may communicate with the members. For example, a teacher may ask his students how fast or how difficult they would like his lectures to be, or a local authority may try to find how much the people want it to spend on a public good. When the decision maker must treat everybody equally, what happens to communication? Does equal treatment encourage or discourage information revelation?

This paper extends the standard cheap talk model of Crawford and Sobel (1982, hereafter CS) where a single rational sender of a message communicates directly with the receiver, and studies communication subject to various constraints such as anonymity, overconfidence, and equal treatment of multiple senders. We identify a remarkably similar strategic feature common to these settings and show that it can not only change the structure of informative equilibria significantly compared with that of CS (and many other related models) but also improve information transmission when the level of conflict between communicating parties is large. Moreover, we demonstrate that our framework can be very closely linked to models in the literature on noisy communication, where a message in one-to-one communication is assumed to be randomized according to a certain garbling mechanism.

As a starting point of our analysis we show that, with the communication features we have introduced, the action by the receiver (uninformed decision maker) becomes less sensitive to the message from a sender (informed agent), compared with CS. In the case of anonymous communication, since the receiver does not know who has sent which message, it is not beneficial for her to differentiate between senders according to the messages. That

is, the receiver takes the same action towards every sender. As a result, the receiver's action towards a particular sender's message becomes less sensitive because the receiver will take into account all messages when deciding on her action for a particular sender.

Clearly, if it is possible for the receiver to commit to treating every sender equally in a multiple sender environment, it will have the same effect on the receiver's response to messages as anonymity. Therefore, anonymity can be considered as one of possible commitment devices for equal treatment.

Equal treatment may not be a constraint to be imposed but inherent in certain communication environments, such as revelation of preferences for public goods. Suppose that a decision maker asks the agents about the quality or quantity of a public good to be supplied, while each agent has a different preference for the public good. Whether or not the agents are anonymous, the decision maker is restricted to supply the same amount of the public good consumed by all agents concerned. As a result, the sensitivity of the supply of the public good to a particular message from an agent becomes weaker as the number of the agents becomes larger. Therefore, the analysis of anonymous communication can be directly applied to study information transmission in public good provision.

In single sender settings overconfidence and garbling also make the receiver's action less sensitive to a message. If there is a possibility that the message does not convey any information about the true state (sender type), the receiver will put less weight on the message and her decision is based more on prior belief. This implies that, if modelled appropriately, overconfidence and garbling may be analyzed in the same framework as communication under anonymity or communication in public good provision.

In CS, what makes a message less credible than truthful revelation is the presence of the sender's systematic bias that reflects a conflict of interest between the sender and the receiver. That is, in any state the sender wishes to induce a higher (or lower) action than the receiver and no types reveal truthfully because doing so always leads to a lower (or higher) action than the sender wants. Consequently, the informative equilibria of CS are characterized by a partition of the sender's type space into a finite number of intervals, where the types of sender in the same interval induces the same action.

In the informative equilibria of our model the type space is also partitioned into intervals. However, in our model there is another source of information distortion: reduced sensitivity to a message. This gives rise to some interesting characteristics in the sender's equilibrium strategies that are not found in CS.

First, while the bias in CS leads to either overstatement (in the case of upward bias) or understatement (downward bias), the reduced sensitivity causes *exaggeration*, where incentives to overstate and understate can coexist ex ante (before the sender observes private information). When the receiver's action is less sensitive to a message, the sender

has incentives to report a message recommending an extremely low (high) action even if he actually wishes to see only a moderately low (high) action. As a result, even if individual conflict is absent, the sender has incentives to *exaggerate* his type, in that he is tempted to overstate the *difference* between his type and the average type.

Second, when the receiver's action is less sensitive, there may be a type of sender who may reveal truthfully even in the presence of bias. A remarkable consequence of the existence of the truth-telling type is that as the sender's type becomes closer to the truth-telling type, he is more willing to report accurate information. This also implies that, unlike CS, there can be an infinite number of intervals in the neighbourhood of the truth-telling type.

Finally, the individual conflict of interest (systematic bias) and incentives to exaggerate caused by weak response may partly offset each other, and when the level of conflict is large, the introduction of a constraint that leads to weak response may improve information transmission and welfare. When the level of conflict is high, revealing truthfully is highly likely to induce an action unfavourable to the sender. When the receiver's action is less sensitive to a message, the influence of a truthful message on the receiver's action is attenuated, which means that the costs of revealing information may be lower. We illustrate that the informational advantage of weak response is common to anonymity, equal treatment, overconfidence, and garbling.

1.1 Relation to the Literature

It is already known in the literature that the introduction of garbling or randomness in messages may facilitate information transmission even in the presence of large conflict of interest between communicating parties. Myerson (1986) and Forges (1986) have shown that multiple stage communication through a neutral third party may make information transmission possible even when the level of conflict is high, although their analyses abstract from specific ways in which such a third party should be involved. Krishna and Morgan (2004) and Mitusch and Strausz (2005) proposed particular garbling mechanisms to improve information transmission, whereby a "mediator" randomizes the message from the sender. In contrast Ganguly and Ray (2006) focus on situations where the introduction of garbling cannot improve welfare. In this line of research a recent paper by Blume and Board (2006) has obtained a strong welfare result: they have shown that in the uniform-quadratic setting of CS if a certain garbling device is introduced it is possible to construct an equilibrium that Pareto dominates any equilibrium in CS as long as the level of conflict is not too large.

One of our contributions here is to show that the framework for studying noisy communication extends much more widely than to garbling of messages which, though typically

interpreted as what a "mediator" does, may not be easily introduced in practice. In particular, the same framework extends to other types of information transmission such as communication with anonymity, overconfidence, and in public good provision.

Another important contribution of this paper is to show that reduced sensitivity to a message can change the structure of informative equilibria significantly. We characterize informative equilibria under a certain noise structure (for which garbling only one possible interpretation) and examine important strategic features produced by the noise. In particular, we are able to demonstrate how the existence of the truth-telling type can affect the structure of informative equilibria, and how reduced response to a message gives rise to incentives to exaggerate.

As anonymity and the concept of equal treatment necessarily involve multiple senders, our model is also related to the literature on communication with multiple experts. Krishna and Morgan (2001) study when senders should be consulted simultaneously or independently and Battaglini (2002) shows that when there are multiple senders with different biases and the state space is multidimensional, full revelation can be achieved for an arbitrarily large conflict of interest. A common feature of these papers is that the senders observe the same or correlated states of nature while they have different biases. Our model is closer to Austen-Smith (1993) and Wolinsky (2002) where senders observe independent signals (types). However, while we assume that the sender types are distributed continuously, in their models the individual types and messages are assumed to be binary. Because of the binary structure incentives to exaggerate cannot be fully incorporated into their models. Austen-Smith (1993) focuses on the comparison between simultaneous and sequential reporting, and Wolinsky (2002) considers information sharing between senders. Thus the questions they address are also different from ours. As we will see later, in our multiple sender setting the senders may be better thought of as agents in a social choice problem rather than as experts, although in our single sender context the sender will be interpreted as an expert.

Overconfidence has been attracting much attention from physiologists and economists. The literature on judgement under uncertainty has found that people tend to be overconfident about their judgement, in that their subjective probability distributions are too tight (Kahneman et al., 1982). Overconfidence has been found in various professions such as clinical psychologists (Oskamp, 1965), lawyers (Wagenaar and Karen, 1982), and policy experts (Tetlock, 1999). The implications of overconfidence for economic choices and especially for financial markets have been studied recently by numerous researchers (e.g. Kyle and Wang, 1997; Gervais and Odean 1998; Daniel, Hirshleifer and Subrahmanyam, 1998; Scheinkman and Xiong, 2003). This paper examines the nature of communication with an overconfident expert (whatever role the expert plays) rather than pricing or market

returns.

The literature on public good provision has been studying mechanism design problems where agents reveal their preferences (partially or fully) by contributing to or voting for the provision of a public good (Palfrey and Rosenthal, 1984; Bagnoli and Lipman, 1989; Ledyard, 1995). The decision maker is typically assumed to be a mechanism designer and able to commit to a mechanism. This paper offers an alternative approach to problems in public good provision by assuming that the agents communicate with the decision maker who cannot commit to a mechanism. In other words, the decision maker makes her decision strategically after hearing or reading the messages, which seems to be the case in many practical situations. Specifically, our model can be interpreted as referring to situations where the decision maker determines the quality/quantity of a public good without costs or imposes the same cost/provision pair on all agents, each of whom has a different and privately known preference.

The structure of informative equilibria we identify is also related to that of Melumad and Shibano (1991) and Alonso, Dessein and Matouschek (2006), who, like us, study cheap talk models with reduced response to the sender type (or a message), and find an equilibrium with the truth-telling type. Unlike our model, Melumad and Shibano (1991) introduce reduced response directly into the receiver's utility function, and they do not derive the most informative equilibrium. Alonso, Dessein and Matouschek (2006) find the equilibrium with an infinite number of intervals in a two sender model where the receiver coordinates her action for each sender. Since coordination requires taking into account messages from both senders, the sensitivity of the receiver's action to a message is reduced as a result of her best response. Our model shares some characteristics in the receiver's action and the most informative equilibrium. However, since Alonso, Dessein and Matouschek (2006) focus on the issue of coordination and communication, they do not consider systematic bias which has been the centre of attention in the cheap talk literature since CS. Not only do we show that the interaction between the systematic bias and exaggeration has important strategic and welfare implications, but we also show that our simple yet general framework can be applied to many distinct situations, including both single and multiple agent settings.

This paper proceeds as follows. Section 2 introduces the model where utility functions are quadratic and a sender's type is uniformly distributed. Section 3 characterizes the equilibria of the model. Section 4 compares the players' expected utilities in our constrained communication settings and those in CS. Section 5 considers non-quadratic utilities and shows that the introduction of the constraints extends the possibility of communication in a more general setup. Section 6 concludes.

2 Model

Consider first the standard "uniform-quadratic" model of CS. A sender who has private information about his type (or state of nature) θ communicates with a receiver. The sender's utility function is $U^S = -(y - \theta - b)^2$ and the receiver's is $U^R = -(y - \theta)^2$, where y denotes the receiver's action. The sender's type θ is uniformly distributed on $[0, 1]$ and $b \geq 0$ represents the sender's bias or the level of conflict. Before the receiver takes her action, the sender can report a costless message m . The receiver updates her belief on θ according to the message. The first order condition gives her best response conditional on the message,

$$y = E[\theta \mid m]. \quad (1)$$

CS have shown that, for $b > 0$, the perfect Bayesian equilibria of this game are such that, the type space is divided into a finite number of intervals and any types in a particular interval induces the same action. There are multiple equilibria with any integer j such that $1 < j < J$ where J is the largest number of intervals supported in equilibrium. CS have also shown that in this setting J is non-increasing in b . If $b = 0$ both parties' interest coincide and truthful communication is possible, that is, $y = \theta$ for all θ . The influence the sender's message has on the receiver's action is captured in (1). Hence, the sender's equilibrium strategy must be a best response to (1).

Let $y_S(m)$ denote the receiver's (expected) action from the sender's viewpoint, conditional on the message he has reported. In CS, we have $y_S(m) = E[\theta \mid m]$: the action conditional on the message appears the same for both the sender and the receiver. In the following we will extend the basic framework in some different ways, to study communication settings constrained by anonymity, equal treatment, overconfidence and garbling. Let us consider anonymous communication first. As we have suggested earlier, when the receiver chooses an action based on messages from many senders, from a sender's viewpoint his message has less influence on the receiver's action compared with the standard one-to-one communication. As we will demonstrate shortly, the receiver's expected action *from a sender's viewpoint* is given by the following form:

$$y_S(m) \equiv \gamma E[\theta \mid m] + \frac{1}{2}(1 - \gamma), \quad (2)$$

where $0 < \gamma < 1$. It is easy to see that, compared with (1), the receiver puts less weight on the sender's message because it is weighted at $\gamma < 1$. Moreover, in expected terms, the action is biased towards the unconditional expectation of the sender's type $1/2$. Later we will derive (2) as the receiver's best response to the sender's message in several different settings.

Before deriving (2) formally, let us briefly comment on the implications of the reduced response to the sender's message. Although the change in the best response from (1) to

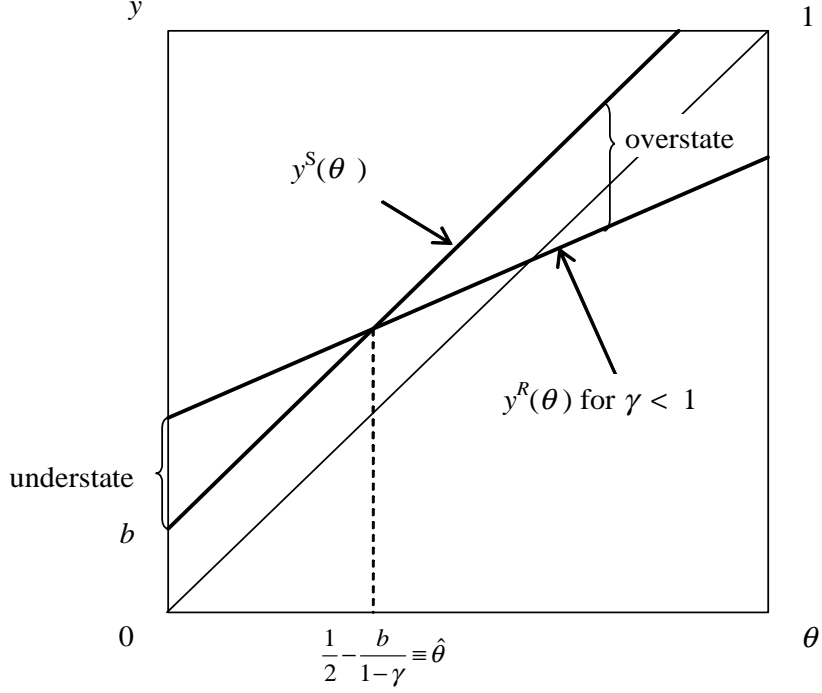


Figure 1: Sender's desired action and receiver's best response

(2) may look innocuous, it may have a great impact on a sender's strategy. First, when the receiver's best response is given by (2), there may be a "truth-telling type", the type of sender who may report his true type in equilibrium even for $b > 0$. Suppose that the sender's type is θ . The sender's desired action is $\theta + b \equiv y^S(\theta)$, while the receiver's best response with θ is, according to (2), $\gamma\theta + \frac{1}{2}(1 - \gamma) \equiv y^R(\theta)$. Hence, If the sender's type is such that

$$\begin{aligned} y^S(\theta) &= y^R(\theta) \text{ or} \\ \theta &= \frac{1}{2} - \frac{b}{1 - \gamma} \equiv \hat{\theta}, \end{aligned} \quad (3)$$

the sender may induce his (expected) desired action $\theta + b$ by reporting truthfully. Hence, if $\gamma < 1$ and b is not too large, there may exist $\hat{\theta} \in [0, 1]$ that satisfies (3). The sender's desired action and the receiver's best response for given θ are illustrated in Figure 1 where the horizontal axis denotes the sender's type θ and the vertical axis denotes the receiver's action y . In CS the receiver's best response (1) is given by the 45 degree line, which never coincides with the sender's desired action $y^S(\theta)$. This implies that no type has incentives to tell the truth in CS. In contrast, when the receiver's best response is $y^R(\theta)$, it crosses

$y^S(\theta)$ at $\hat{\theta}$, which implies that the sender's desired action and the receiver's best response coincide at $\hat{\theta}$.

Second, when the receiver's best response is given by (2) the sender may have incentives to exaggerate his type. In CS, $y^S(\theta)$ is always above the 45 degree line for $b > 0$, so that the sender has incentives to overstate his type only. However we have $y^S(\theta) < y^R(\theta)$ ($y^S(\theta) > y^R(\theta)$) if $\theta < \hat{\theta}$ ($\theta > \hat{\theta}$), in which case the sender has incentives to understate (overstate) his type. Hence, unlike CS, the sender can be biased in both directions, depending on his type.

As we will see later, these two features change the structure of informative equilibria significantly. Before characterizing equilibria, however, let us motivate (2) through several different examples of constrained communication.

2.1 Anonymous Communication

Suppose that there is a single receiver who communicates with n senders. The payoff of the receiver is given by von Neumann-Morgenstern utility $-\sum_{i=1}^n (y_i - \theta_i)^2$, where y_i is the receiver's action towards sender i and θ_i is his type. In other words, the receiver determines n -dimensional action $\mathbf{y} = [y_1, y_2, \dots, y_i, \dots, y_n]$. Sender i 's von Neumann-Morgenstern utility is given by $-(y_i - \theta_i - b)^2$, where b represents the ex-ante individual bias that is assumed to be symmetric across all senders. We also assume $b \geq 0$ without loss of generality. θ_i is private information to sender i , and independently and uniformly distributed on $[0, 1]$. Before the receiver chooses her action, each sender reports a message m_i on his type, independently and simultaneously. We focus on symmetric sender strategies, that is, any senders with the same type follow the same strategy.

Under anonymity, the receiver gets n messages before she chooses $y_1, y_2, \dots, y_i, \dots, y_n$, but she cannot tell which sender has sent a particular message. In other words, every message corresponds to a particular sender with probability $1/n$. Therefore, although the receiver can in principle take a different action towards each sender, the receiver's best response conditional on the messages is to treat every sender equally, or $y_1 = y_2 = \dots = y_i = \dots = y_n$.

To motivate this setup, consider a situation where a manager in a firm must allocate tasks to his workers. Each worker has a privately known ability or idea about what he wants to do, but they all would like to have an easier task than the receiver would want them to do. If each worker communicates with the manager directly, the workers may not reveal their information truthfully because they may fear that they might be assigned too hard a task. In this situation, the manager may choose to communicate with the workers anonymously, through anonymous report forms or a worker representative. In fact, as we will show later, when the level of individual conflict is large, it may be the case that

messages are completely uninformative in one-to-one communication but some information may still be transmitted in anonymous communication.

Let us consider the receiver's action from a sender's perspective. The receiver's best response to the n messages she receives is given by

$$y_i = E[\theta_i] = \frac{1}{n} \sum_{i=1}^n E[\theta_i | m_i]. \quad (4)$$

From sender i 's viewpoint, therefore, the expected action by the receiver conditional on the sender's own message is

$$y_{Si}(m_i) = \frac{1}{n} E[\theta_i | m_i] + \frac{n-1}{n} E[E[\theta_i | m_i]],$$

since sender i does not observe other senders' types or messages. Using the fact that the expected value of a conditional expectation is the prior expectation, we have

$$y_{Si}(m_i) = \frac{1}{n} E[\theta_i | m_i] + \frac{n-1}{n} \times \frac{1}{2}. \quad (5)$$

Note that (5) is a special case of (2) where $\gamma = 1/n$. Hence, a perfect Bayesian equilibrium of this game can be characterized by deriving each sender's best response to (2).

2.2 Equal Treatment/Public Good Provision

The above discussion on anonymous communication implies that the essential feature that leads to (5) is equal treatment of senders. That is, to the extent that the receiver can commit to equal treatment, it will have the same effect on the sender's strategy as anonymity.

In certain environments equal treatment is implied in the receiver's action. Consider non-anonymous communication with n agents (senders) where the decision maker (receiver) determines the provision of a public good y . For instance, a high y may be a large amount of the public good with heavy tax burden. Alternatively, y can be thought of as the pace of a lecture in a classroom of n students with different abilities. The difference from anonymous communication is that equal treatment is inherent in the receiver's action because of the nature of the public good to be supplied. In other words, the receiver chooses a scalar y that determines the utilities of all agents.

Each of the agents has a different preference for y and the utility of an agent is given by $-(y - \theta_i)^2$. Unlike the case of anonymous communication, in this context it would be appropriate to assume $b = 0$, so that there is no individual conflict between the decision maker and each agent. The decision maker maximizes the sum of the agents' utilities $-\sum_{i=1}^n (y - \theta_i)^2$, but she cannot commit to a mechanism. The agents send messages to the

decision maker before she chooses y . Under these assumptions, since the receiver's maximization problem is identical to that of the anonymous communication case, the receiver's best response is given by (4). Thus the expected action from a sender's viewpoint the receiver's action from sender i 's perspective is (2). Therefore, we can examine communication regarding the provision of a public good by a benevolent decision maker in the same way as anonymous communication.

2.3 Overconfident Expert

So far we have assumed that there are multiple senders, but for this and the next application we show that (2) can be motivated even in settings with a single sender. In the previous applications we have maintained the assumption that the sender observes his type correctly. What if the sender is overconfident about what he has observed, when the type the sender observes may be wrong? This situation corresponds to a class of behavioural attributes called *overconfidence in the precision* of one's information. Specifically, we assume that the sender observes pure noise with a certain probability, but he always believes that his observation is correct.

Let p be the probability that the sender observes the true type, and $1 - p$ be the probability that the type the sender observes is pure noise, which is uniformly distributed on $[0, 1]$ independently of the true type. We assume that p is common knowledge. Clearly, $p = 1$ in CS. The notion of overconfidence is captured by the assumption that the sender believes that the type he observes is always correct.¹ From the receiver's viewpoint the message reflects the true type only with probability p , and otherwise the message is completely uninformative about the true type. Hence, we have the receiver's best response given the message

$$y_R(m) = pE_s[\theta \mid m] + (1 - p)\frac{1}{2}, \quad (6)$$

where $E_s[\theta \mid m]$ is the expected value of θ conditional on the message, provided that the sender's observation is correct. Note that from the overconfident sender's viewpoint the receiver's action is also given by (6). Since the sender believes that his observation is always correct, the receiver's action in communication with the overconfident sender takes the same form as (2), where $\gamma = p$.

¹It is possible to interpret this set-up in terms of the receiver's *underconfidence* in the sender's observation. Suppose that the sender always observes a correct type but the receiver (due to some irrationality) believes that the sender observes the true state only with probability q and otherwise observes a noise. It is easy to check that the equilibrium strategies are equivalent to the case of an overconfident sender. The only difference lies in interpretation of welfare.

2.4 Garbling

As we have seen earlier, it is known in the literature that the introduction of randomization in messages may facilitate information transmission particularly when the level of conflict is large. We show that (2) can be derived through a garbling device too. Suppose that with probability $q < 1$ the message from the sender is delivered to the receiver as it has originally been reported. However, with probability $1 - q$ the message is randomized according to the ex ante distribution of messages induced in equilibrium. We assume that q is common knowledge. Naturally $q = 1$ in CS. This garbling device has the analytically convenient feature that upon receiving a message the receiver cannot update her belief on whether the message is from the sender himself or generated by randomization. In other words, whatever the message is, the posterior probability that it is "genuine" is also q .

To see how this randomization device works, consider an informative equilibrium with two intervals where the message induces either "low" or "high" action. Suppose that ex ante (before the sender learns his type) a message recommending "low" is sent with probability a and a message recommending "high" is sent with probability $1 - a$ (in other words, with probability a the sender's type is such that he recommends "low"). Suppose further that the sender type is such that he recommends "low". He reports a message that recommends "low", but with the garbling device the message is delivered to the receiver without randomization only with probability q . With probability $1 - q$ the message is randomized, so "low" and "high" are recommended with probability a and $1 - a$ respectively. Overall, given that the sender recommends "low" action, the probability that this action is actually induced is $q + (1 - q)a$.

From the receiver's perspective, the message she receives is "genuine" only with probability q . Otherwise the message is the one that was generated by the randomization device. In addition, since the distribution of messages with randomization is identical to the equilibrium distribution of messages the sender reports conditional on the delivered message, the receiver cannot update the belief on whether the message has been generated by randomization. Let m_S and m_R be the message the sender has actually reported and the message delivered to the receiver, respectively. The receiver's best response to a message is given by

$$y_R(m_R) = qE[\theta \mid m_S = m_R] + (1 - q)\frac{1}{2}. \quad (7)$$

$E[\theta \mid m_S = m_R]$ is the expected θ given that the message is the one originally reported by the sender, which is the case with probability q . From the sender's viewpoint, this implies that if the original message is delivered to the receiver

$$y_S(m_S) = qE[\theta \mid m_S] + (1 - q)\frac{1}{2}. \quad (8)$$

Note that if the message is garbled the sender's report has no influence on y . Therefore, as in the previous applications introduced above, the sender's equilibrium strategy with the garbling mechanism can be derived by considering a best response to (2), where $\gamma = q$.

The garbling device we study here is closely related to one studied by Blume and Board (2006), which also leads to (8) in the uniform-quadratic setting if the sender's strategy is specified appropriately. In their model when garbling occurs the message delivered to the receiver is uniformly distributed on $[0, 1]$. Blume and Board (2006) also consider sender strategies that allow the receiver to update her belief on whether the message is from the sender or from the uniform distribution, in which case (8) does not hold.

3 Equilibrium

In the following we study the sender's strategy with respect to (2), the receiver's best response to the message. In the standard setting, CS have shown that, if the receiver's ideal action and that of the sender never coincide, any perfect Bayesian equilibrium takes a partitioned form, where the type space is divided into a finite number of intervals. If the receiver's best response is given by (1) as in CS, both parties' desired actions differ for any sender types: while the receiver's optimal action is θ , the sender's is $\theta + b$.

Although we will basically follow the equilibrium characterization due to CS, we require one important modification to characterize equilibria when the receiver's best response is (2). The modification concerns the difference between a sender's desired action and that of the receiver for a given sender type. As we have already seen in (3), if the receiver's best response is given by (2), as long as b is not too large there exists a truth-telling type $\hat{\theta}$, whose desired action (in expected terms) coincides with that of the receiver from the sender's viewpoint.

Thus we will identify two classes of equilibria in this game; namely T-equilibria where the truth-telling type exists and induces the unique action for this type, and N-equilibria where there is no truth-telling type or the truth-telling type exists but the truth-telling type induces the same as his neighbouring types (i.e. he sticks to a partitioned strategy). As we will see below, N-equilibria are characterized exactly in the same way as CS. However, for T-equilibria an additional condition is required to satisfy incentive compatibility.

Let us first consider N-equilibria. Let \underline{a} and \bar{a} be two points in $[0, 1]$ such that $\underline{a} < \bar{a}$. Suppose that the sender observes $\theta \in [\underline{a}, \bar{a}]$. From the assumption that θ is uniformly distributed, we have the conditional expectation of θ

$$E[\theta \mid \theta \in [\underline{a}, \bar{a}]] = \frac{\underline{a} + \bar{a}}{2}.$$

Hence, according to (2), we have the receiver's action from the sender's viewpoint given

this belief

$$\begin{aligned}
\bar{y}_S(\underline{a}, \bar{a}) &= \gamma E[\theta \mid \theta \in [\underline{a}, \bar{a}]] + (1 - \gamma) \frac{1}{2} \\
&= \gamma \frac{\underline{a} + \bar{a}}{2} + (1 - \gamma) \frac{1}{2}.
\end{aligned} \tag{9}$$

Note that except in the case of an overconfident sender, the receiver's action is a random variable from the sender's viewpoint. However, the randomness is caused only by the other senders in anonymous communication and public good provision, and by the randomization device in communication with garbling. Hence, the variance of the receiver's action is independent from the sender's strategy (message). Since the utility functions are quadratic, we can focus our attention on the expected value of the receiver's action from the sender's viewpoint, y_S , in order to characterize equilibrium sender strategies.

Proposition 1 Suppose that the receiver's best response is given by (2). Then there exists a positive integer $J(b)$ such that, for every J with $1 \leq J \leq J(b)$, the following sender strategy $q(m \mid \theta)$ supports at least one N-equilibrium:

$$q(m \mid \theta) \text{ is uniform on } m \in [a_j, a_{j+1}] \text{ if } \theta \in (a_j, a_{j+1}). \tag{10}$$

$$\begin{aligned}
-(\bar{y}_S(a_{j-1}, a_j) - \theta - b)^2 &= -(\bar{y}_S(a_j, a_{j+1}) - \theta - b)^2 \\
&\text{if } \theta = a_j, \text{ for } j = 1, \dots, J - 1.
\end{aligned} \tag{A}$$

$$a_0 = 0, a_J = 1. \tag{11}$$

Further, all other equilibria have relationships between θ and the receiver's induced choice of \bar{y}_S that are the same as those in this class for some value of J with $1 \leq J \leq J(b)$.

Proof. See Theorem 1 in CS. In CS $\gamma = 1$ but the same method of proof applies for N-equilibria as long as $0 < \gamma \leq 1$. ■

With the uniform-quadratic setup, the characterization of N-equilibria given (2) is essentially the same as that of informative equilibria in CS. The intervals are determined by the "no arbitrage" condition (A), which says that in equilibrium when sender i 's type falls on the boundaries of intervals a_j , he must be indifferent between two associated actions (from the sender's viewpoint) $\bar{y}_S(a_{j-1}, a_j)$ and $\bar{y}_S(a_j, a_{j+1})$.

Since the utility function is symmetric with respect to y_S and the sender cannot influence $\text{var}(y)$, for (A) to be true, it must be that the expected value of the receiver's action that

the sender with type a_j wishes to induce, $a_j + b$, lies exactly halfway in between these two expected actions induced by two different messages

$$a_j + b = \frac{\bar{y}(a_{j-1}, a_j) + \bar{y}(a_j, a_{j+1})}{2}.$$

Hence from (9) we have

$$a_j + b = \gamma \left[\frac{a_{j-1} + a_j}{4} + \frac{a_j + a_{j+1}}{4} \right] + (1 - \gamma) \frac{1}{2},$$

which gives a second-order difference equation

$$\gamma a_{j+1} - (4 - 2\gamma)a_j + \gamma a_{j-1} = 4b + 2\gamma - 2. \quad (12)$$

Substituting $a_0 = 0$ and $a_J = 1$, we can solve the J simultaneous equations with J unknown variables in (12) to obtain the exact N-equilibrium partition that corresponds to the particular γ , b and J . In other words, any equilibrium partition must satisfy (12). From (12) we obtain the following example.

Example 1 Suppose that $\gamma = \frac{1}{2}$ and $b = 0$. Then the partition $a_0 = 0$, $a_1 = \frac{5}{12}$, $a_2 = \frac{1}{2}$, $a_3 = \frac{7}{12}$, $a_4 = 1$ constitutes an N-equilibrium.

Notice that the length of an interval is narrower as it becomes closer to $\frac{1}{2}$. This means that a message is more credible when it is moderate (closer to $\frac{1}{2}$), and less credible when it is extreme. Intuitively, since the receiver puts less weight on the message, even with $b = 0$ the sender has incentives to exaggerate his type, in that he overstates the difference between his type and the average type $\frac{1}{2}$. If a sender's type is below (above) the average, he reports an even lower (higher) type than he actually is. Therefore, the equilibrium partition above takes into account the tendency to exaggerate.

Interestingly, the above partition does not support a T-equilibrium, where the truth-telling type induces the unique action for him only. To see this, note that since $b = 0$ we have the truth-telling type $\hat{\theta} = \frac{1}{2} (= a_2)$. Suppose that the receiver's belief is such that the truth-telling type reveals truthfully. If $\theta = \frac{1}{2}$ we have the corresponding action $\bar{y}_S(\frac{1}{2}) = \frac{1}{2}$. Let us consider a sender with $\theta_i = \frac{1}{2} + \epsilon$ for some small $\epsilon > 0$. If he sends a message according to the proposed partition, he induces $\bar{y}_S(\frac{1}{2}, \frac{7}{12}) = \frac{13}{24} \approx 0.542$. On the other hand if he mimics the truth-telling type, $y_S = \frac{1}{2}$. Since the sender's ideal action is $\frac{1}{2} + \epsilon$, for small enough ϵ he prefers mimicking the truth-telling type. Hence, the partition in the Example does not support a T-equilibrium.

This implies that T-equilibria cannot simply be characterized by the "no arbitrage" condition (A). What condition must be satisfied for a partition given by (A) to support a T-equilibrium? The problem with supporting T-equilibria is that no other types than $\hat{\theta}$

should benefit from inducing $\bar{y}_S(\hat{\theta})$. In other words, the sender with type $\theta = \hat{\theta} + \epsilon$ must not induce $\bar{y}_S(\hat{\theta})$, but rather his message must induce $\bar{y}_S(a_j, a_{j+1})$ such that $\hat{\theta} + \epsilon \in [a_j, a_{j+1})$. For this to be the case, it must be that the difference between the sender's ideal action $y^S(\hat{\theta} + \epsilon)$ and the action induced by his message is smaller when he follows the partitional strategy:

$$\begin{aligned} y^S(\hat{\theta} + \epsilon) - \bar{y}_S(\hat{\theta}) &\geq \left| y^S(\hat{\theta} + \epsilon) - \bar{y}_S(a_j, a_{j+1}) \right| \\ \Rightarrow \hat{\theta} + \epsilon + b - (\hat{\theta} + b) &\geq \left| \hat{\theta} + \epsilon + b - \bar{y}_S(a_j, a_{j+1}) \right| \\ \Rightarrow 2\epsilon &\geq \bar{y}_S(a_j, a_{j+1}) - (\hat{\theta} + b) \geq 0. \end{aligned} \tag{13}$$

This implies that as long as a partition determined by (A) also satisfies (13), the partition for $[0, 1] \setminus \{\hat{\theta}\}$ with $\hat{\theta}$ telling the truth supports a T-equilibrium. Note that any type such that $\theta_i > \hat{\theta} + \epsilon$ does not mimic $\hat{\theta}$ if $\theta_i = \hat{\theta} + \epsilon$ does not, because $\frac{\partial^2 U^S}{\partial y \partial \theta} > 0$ implies that the utility of $\theta_i > \hat{\theta} + \epsilon$ when inducing $\bar{y}_S(\hat{\theta})$ (by mimicking $\hat{\theta}$) is lower than that of $\theta_i = \hat{\theta} + \epsilon$. Therefore to see whether (13) is satisfied in a partition given by (A), it suffices to consider an infinitesimally small ϵ .

Clearly there are only two cases where (13) is satisfied. One is the case in which $\hat{\theta} + \epsilon \in [a_j, a_{j+1})$, $a_j < \hat{\theta} < a_{j+1}$ for some j and

$$\frac{a_j + a_{j+1}}{2} = \hat{\theta}.$$

If this is the case, $[a_j, a_{j+1})$ is an interval with some positive length that has $\hat{\theta}$ exactly in the middle. The other is the case where $\hat{\theta} + \epsilon \in [a_j, a_{j+1})$ and $a_j, a_{j+1} > \hat{\theta}$ such that the distance between a_j and a_{j+1} is infinitesimally small and the interval is arbitrarily close to $\hat{\theta}$. This second case implies that in a T-equilibrium there can be an infinite number of intervals in the neighbourhood of $\hat{\theta}$. Before we derive the equilibrium with an infinite number of intervals, let us consider the following example of a T-equilibrium with a finite number of intervals.

Example 2 Suppose that $\gamma = \frac{1}{2}$ and $b = 0$. Then the partition $a_0 = 0$, $a_1 = \frac{3}{7}$, $a_2 = \frac{4}{7}$, $a_3 = 1$ supports both T-equilibrium and N-equilibrium.

Since the partition in this Example is derived from (12), clearly it supports an N-equilibrium. In addition, introducing truth-telling for $\hat{\theta} = \frac{1}{2}$ to this partition does not change the structure of the N-equilibrium because both the truth-telling type and any other types such that $\theta_i \in [\frac{3}{7}, \frac{4}{7})$ induce the same action $y_S = \frac{1}{2}$.

Is full revelation possible in this above example where $b = 0$? In CS, full revelation can be supported in equilibrium when $b = 0$. However, in communication subject to the constraints we consider, this is not the case. In fact, full revelation is not an equilibrium for any b .

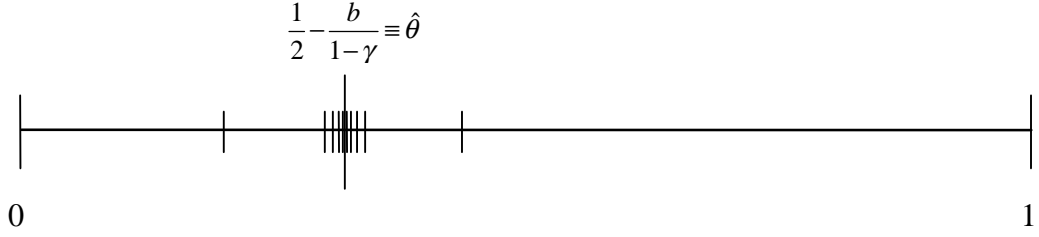


Figure 2: Most informative equilibrium

Proposition 2 If $\gamma < 1$, full revelation is not a perfect Bayesian equilibrium in anonymous communication for any b .

Proof. Suppose that the receiver's belief is such that the sender reveals truthfully ($m = \theta$ for all θ). From the receiver's strategy (2), sender i 's best response m^* is to induce the desired expected action

$$\begin{aligned}\theta + b &= \gamma m^* + \frac{1}{2}(1 - \gamma) \text{ or} \\ m^* &= \frac{\theta + b - 1/2}{\gamma} + \frac{1}{2}.\end{aligned}$$

Hence, $m^* = \theta$ only if $\theta = \frac{1}{2} - \frac{b}{1-\gamma} \equiv \hat{\theta} \in [0, 1]$, $\theta = 0$, or $\theta = 1$. Otherwise, $m^* < \theta$ if $\theta < \hat{\theta}$, and $m^* > \theta$ if $\theta > \hat{\theta}$. ■

When the receiver's best response is given by (2), even if $b = 0$ the sender has incentives to exaggerate the difference between the average $\frac{1}{2}$ and his type. In other words, when his type θ is low, revealing truthfully leads to the receiver's (expected) action that is not low enough for the sender. However, if $\theta = \frac{1}{2}$ the sender reveals truthfully because there is no need of exaggeration.

Now let us characterize the most informative equilibrium in which there are as many intervals as possible in the type space. Solving (12) with respect to a_j , we obtain

$$\begin{aligned}a_j &= \hat{\theta} + \frac{\gamma a_1 + 2\hat{\theta}(1 - \gamma - \sqrt{1 - \gamma})}{4\sqrt{1 - \gamma}} \left(\frac{2 - \gamma + 2\sqrt{1 - \gamma}}{\gamma} \right)^j \\ &\quad - \frac{\gamma a_1 + 2\hat{\theta}(1 - \gamma + \sqrt{1 - \gamma})}{4\sqrt{1 - \gamma}} \left(\frac{2 - \gamma - 2\sqrt{1 - \gamma}}{\gamma} \right)^j,\end{aligned}\tag{14}$$

where $\hat{\theta} = \frac{1}{2} - \frac{b}{1-\gamma}$ is the truth-telling type. Let a_J be the terminal point of the partition and $a_J = 1$. Rearranging (14) and letting $J \rightarrow \infty$, we have

$$a_1 \rightarrow 2\hat{\theta} \left(1 - \frac{1 - \sqrt{1-\gamma}}{\gamma} \right) \equiv a_1^*.$$

This implies that as long as $a_1^* > 0$, (14) can generate an infinite number of intervals in equilibrium. Indeed, substituting a_1^* into (14), we obtain

$$a_j = \hat{\theta} - \hat{\theta} \left(\frac{2 - \gamma - 2\sqrt{1-\gamma}}{\gamma} \right)^j. \quad (15)$$

Note that since

$$0 < \frac{2 - \gamma - 2\sqrt{1-\gamma}}{\gamma} < 1 \text{ for } 0 < \gamma < 1,$$

(15) gives a strictly increasing sequence that converges to the truth-telling type $\hat{\theta}$. This converging sequence constitutes a partition in $[0, \hat{\theta})$. Let the sequence of a_j 's obtained by (15) be P_0 .

It remains to obtain the partition in $(\hat{\theta}, 1]$. Let a'_j be a decreasing sequence such that $a'_0 = 1$. Solving (12) with $a'_0 = 1$ and $J \rightarrow \infty$, we have a strictly decreasing sequence that converges to $\hat{\theta}$

$$a'_j = \hat{\theta} + (1 - \hat{\theta}) \left(\frac{2 - \gamma - 2\sqrt{1-\gamma}}{\gamma} \right)^j. \quad (16)$$

This sequence constitutes a partition of $(\hat{\theta}, 1]$ with an infinite number of intervals. Let the sequence obtained by (2) be P_1 . Define $P \equiv P_0 \cup P_1 \cup \hat{\theta}$. Clearly P satisfies (12) and (13), and therefore supports the most informative equilibrium for $\hat{\theta} \geq 0$ or equivalently $b \leq \frac{1}{2(1-\gamma)}$.

From Figure 2 we can see that in the most informative equilibrium there are an infinite number of intervals in the neighbourhood of the truth-telling type $\hat{\theta}$, who induces the unique action for him. The equilibrium partition also takes into account the sender's incentives to exaggerate his type. The length of intervals is narrower as they become closer to $\hat{\theta}$, which implies that a message is more credible when it is closer to the truth-telling type.

Figure 2 suggests that under many circumstances (anonymity, equal treatment, public good provision, overconfidence, and garbling) considered so far, weak sensitivity to a message has just as important implications for the nature of information transmission as the systematic bias b , which has been the centre of attention in the cheap talk literature. In particular, as long as b is not too large the truth-telling type exists under a wider range of parameters, and this can change the structure of informative equilibria dramatically, compared with CS and many models of information transmission where the sole source of informational distortion is b and the sender has incentives to overstate θ (for $b > 0$) only.

In the rest of this section we consider for what value of b informative communication is possible. Let \bar{b} denote the largest bias that can support an informative equilibrium. Since J is non-decreasing in b , in order to find \bar{b} it suffices to consider the condition under which the equilibrium with two intervals can be supported. By solving (12) for $a_0 = 0$, $a_2 = 1$, we obtain $a_1 = \frac{1}{2} - \frac{2b}{2-\gamma}$. In order for this equilibrium to be supported it must be that $a_1 \geq 0$, which implies

$$b \leq \bar{b} = \frac{2-\gamma}{4}. \quad (17)$$

In the uniform-quadratic case of CS it has been shown that an informative equilibrium exists for $b \leq 1/4$, which can be confirmed in our model, by letting $\gamma = 1$. The above condition implies that by introducing $\gamma < 1$ the possibility of informative communication can be extended beyond $b = 1/4$. We have that $\bar{b} \rightarrow 1/2$ as $\gamma \rightarrow 0$. As we will see later in our welfare analysis, this property leads to another important result of this paper: not only may γ change the structure of informative equilibria, but it may also enhance information transmission when b is large.

If the level of conflict b exceeds $1/4$, communication is completely uninformative without anonymity, equal treatment, overconfidence, or garbling. However, when communication is subject to these constraints we may potentially have an informative equilibrium up to $b < 1/2$ and this will prove welfare improving.

A wider possibility of information transmission when b is large comes from the fact that the receiver's response to a message is weaker, as we have seen in (2). The key parameter is γ , the sensitivity of the receiver's action to a message. For example, suppose that θ is very low and that the receiver believes that the sender reveals truthfully. When $\gamma = 1$ the receiver's best response is (1), so by revealing truthfully the sender induces the action $y = \theta$ which is lower than the sender's ideal action $\theta + b$. On the other hand, when $\gamma < 1$ even if the sender reveals θ truthfully, the corresponding action by the receiver may be higher than θ (and closer to $\theta + b$). Hence, for a relatively large value of b , reduced response to the message works as if it reduces the level of conflict.

4 Comparison

Although the constraints on communication considered above share the strikingly similar strategic feature represented by (2), the players' expected utilities in an informative equilibrium differ significantly depending on the particular setting in question. Therefore in this section we consider the multiple sender setting (anonymity/equal treatment) and the single sender setting (overconfidence/garbling) separately.

Before examining when introducing a particular constraint is beneficial, let us briefly consider the issue of multiple equilibria. As in most cheap talk models, there are multiple

equilibria in our model. In particular, there always exists a "babbling" equilibrium where the sender's message is completely uninformative. CS have shown that in the single sender game for given b both the sender and the receiver prefer an equilibrium with more intervals as long as the game satisfies their monotonicity assumption (M).² The equilibrium partition in our model satisfies (M), and some of their results extend to our model.

Proposition 3 In communication subject to any constraint introduced above, namely anonymity, equal treatment, overconfidence and garbling, if we fix γ (i.e. n , p , or q) both the receiver and the sender(s) are better off in an equilibrium with more intervals.

Proof. See Appendix. ■

For the overconfident expert scenario, we can calculate the sender's expected utility based on his ex ante belief that his observation is always correct, or on the ex post realization of his payoff. Proposition 3 applies in both cases.

4.1 Anonymous Communication/Equal Treatment

Let us focus on the most informative equilibria for given b and consider when anonymous communication (and hence imposing equal treatment) is more desirable than CS. Note that without anonymity or commitment to equal treatment the information transmission between the receiver and sender i in our model is equivalent to the single sender game in CS. Therefore we can directly compare the equilibrium outcome of our model and that of CS. Since $\gamma = 1/n$, in this setting (17) implies that information transmission can occur up to $b = 1/2$ as the number of anonymous senders becomes sufficiently large ($n \rightarrow \infty$). In the previous section we have shown that the most informative equilibrium involves an infinite number of intervals for $b \leq \frac{1}{4}$ and communication under anonymity is informative if $b \leq \frac{2-\gamma}{4} = \frac{1}{2} - \frac{1}{4n}$. Hence, if $b \in [\frac{1}{4}, \frac{1}{2} - \frac{1}{4n})$, information transmission can occur even if messages are completely uninformative in CS. The more senders there are, the larger the bias can be for information transmission (and welfare improvement) to occur in equilibrium.

The comparison between the most informative equilibria when $n = 2$ is shown in Figure 3. The horizontal axis represents the bias b and the vertical axis represents the expected utility of the receiver. Since $n = 2$, $U^R = -(y_1 - \theta_1)^2 - (y_2 - \theta_2)^2$. Note that the curve for CS becomes flat for $b \geq 1/4$, indicating that communication is completely uninformative. Both curves are the same for $b \geq 3/8$, where communication is uninformative also in anonymous communication. We can see that the receiver prefers anonymous communication if $b \gtrsim 0.194$. A similar graph can be drawn for the senders, and they also prefer anonymous

²See p.1444 in CS and Appendix of the present paper.

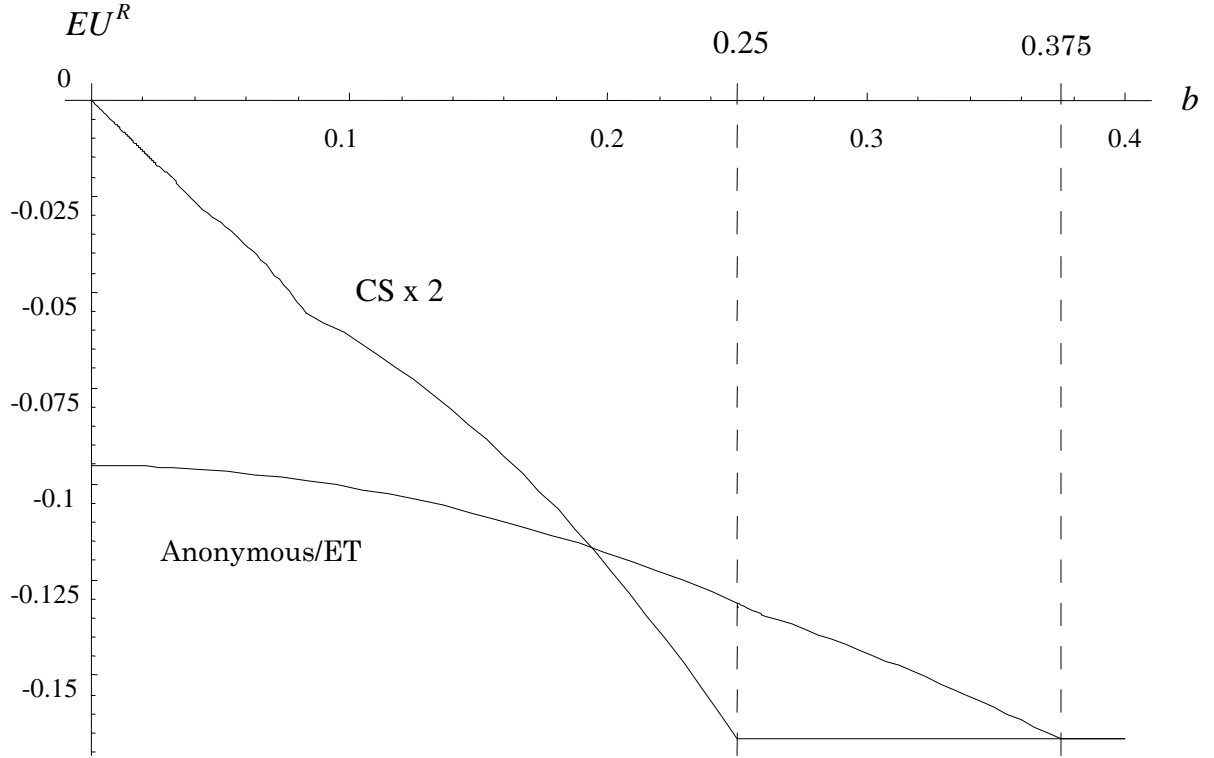


Figure 3: Receiver's expected utility in the most informative equilibrium when $n = 2$

communication when $b \gtrsim 0.194$. When the level of conflict is large, both parties can benefit from anonymous communication.

This result is driven by the trade-off between inflexibility in the receiver's action and information revelation. Imposing equal treatment itself is costly because it prevents the receiver from choosing her action optimally for each sender. However, when b is high reduced sensitivity as a result of equal treatment induces more information revelation. When the level of individual conflict is high, the benefit of information transmission becomes so large relative to CS that it outweighs the cost of inflexibility. Naturally, anonymity can be considered as a commitment device for the receiver to treat every sender equally. If she can commit to equal treatment in some other way, it will also have the same effect on communication as anonymity.

4.2 Garbling

In our model, from the receiver's viewpoint overconfidence and garbling have exactly the same effect on the characteristics of the message she receives: with probability p (overcon-

fidence) or q (garbling) the message is indeed indicative of θ , and otherwise it does not convey any information. By assumption, of course, the receiver does not know whether the message is informative at all.

Let us first look at the model in terms of the garbling scenario and focus on the equilibrium with two intervals, which is analytically convenient and has an especially straightforward interpretation: there are two possible recommended actions, and each recommendation the sender has reported is replaced by the other with certain respective probabilities. Krishna and Morgan (2004) and Mitusch and Strausz (2005) interpret such a randomization mechanism as the role a mediator plays between communicating parties.

In order to see the direct relationship between the level of conflict and the degree of randomization, the receiver's expected utility for given q and b under a large conflict is given in Figure 4, where $U^R = -(y - \theta)^2$. This figure shows that the introduction of garbling can increase the receiver's expected utility, and a similar graph can be drawn for the sender's expected utility too. In CS it is implied that $q = 1$, and there is no informative equilibrium when b is larger than $1/4$. However, when $q < 1$ is introduced there may be an informative equilibrium up to $b < 1/2$ and it Pareto dominates the uninformative equilibrium. If q can be chosen so as to maximize both parties' expected utilities, it will take into account the trade-off between information loss due to garbling and information revelation by the agent. When q is lower the message from the sender is more likely to be garbled and this makes the receiver's action even less sensitive to the received message. While the possibility that the message is garbled is costly, reduced sensitivity encourages information revelation which is beneficial in itself. The inverted U-shaped curves in Figure 4 capture this intuition.

With a related but more strategically flexible garbling device than ours, Blume and Board (2006) have shown that it is possible to construct an equilibrium that Pareto dominates all CS equilibria for almost any $0 < b < 1/2$, although a simple interpretation as above is not available because the way Blume and Board (2006) specify the sender strategy is more complex.³

4.3 Overconfidence

Given the strategic similarities, overconfidence can also be regarded as a way to overcome an expert's intrinsic bias. As we have discussed earlier, apart from being overconfident an expert may well be biased towards a particular direction. For example, when asked to

³Blume and Board (2006) assume that when randomized the message delivered to the receiver is distributed uniformly on $[0, 1]$. Depending on sender strategy (i.e. how he randomize his message given his type), this garbling device allows for a far wider range of equilibrium strategies than the ones we consider in this paper. In particular, an equilibrium strategy can be such that, upon receiving a message, the receiver can update his belief on whether the message is from the sender himself or from the uniform distribution.

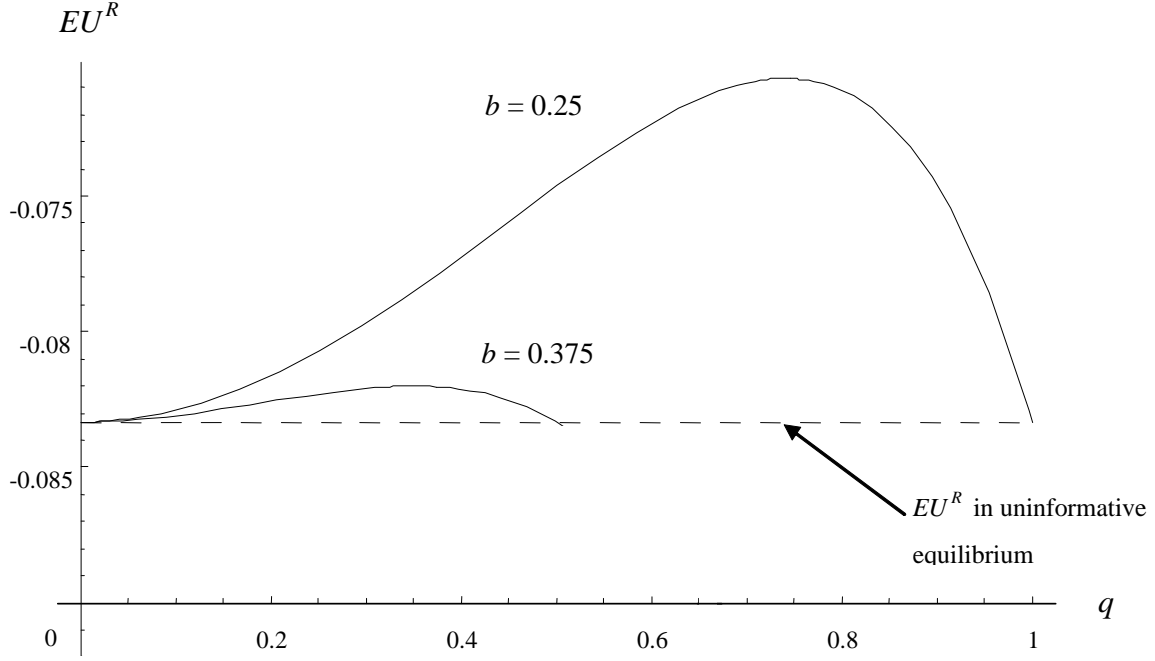


Figure 4: Receiver's expected utility in equilibrium with two intervals

assess the profitability of a potential project an expert may be tempted to overstate the profitability, especially when he has been involved in planning and already invested substantially in the project. A policy consultant may also be biased depending on his political stance. When the bias is large, communication with such experts becomes less informative or possibly uninformative at all. However, our analysis suggests that overconfidence in their expertise, which is generally thought of as a negative factor, may actually help to encourage information revelation.

If a sender observes the true state only with probability p but he correctly recognizes p (i.e. he is not overconfident), the analysis is analogous to CS. In particular, there will never be incentives to exaggerate or a truth-telling type (See Appendix). However, the sender's ideal action given that he has observed θ is now

$$y^S(\theta) = p\theta + (1 - p)\frac{1}{2} + b.$$

In Appendix, we show that if the sender recognizes p correctly

$$\bar{b} = \frac{p}{4}.$$

Therefore, the largest bias that allows communication to occur is lower than $1/4$. The lack of expertise (low p) makes the problem caused by the sender's bias even worse, but

overconfidence may extend the possibility of communication and improve welfare.

5 General Setting

So far we have focused on quadratic utility functions with uniformly distributed sender types, in order to illustrate the characteristics of various communication settings such as anonymity, public good provision, overconfidence and garbling. In particular, we have shown that introducing these features in communication may improve information transmission, especially when the level of conflict is large. In this section, we show that this property holds more generally than the uniform-quadratic setting we have considered so far.

Let a sender i 's von Neumann-Morgenstern utility be $U^{S_i} = U(y, \theta_i, b)$ and that of the receiver be $U^R = \sum_{i=1}^n U(y, \theta_i, 0)$, where $U_1 = 0$ for some y (so there is a unique maximum with respect to y), and $U_{11} < 0$, $U_{12} > 0$, and $U_{13} > 0$. We assume that θ_i has a continuous density function on $[0, 1]$, but the distribution does not have to be uniform. However, we also assume that the utility functions and the type distribution are such that, if a rational sender and the receiver communicate directly as in CS, any equilibrium partition satisfies their monotonicity condition (M). If there are multiple senders, types are independently and identically distributed. Let \bar{b} be the highest level of bias for which an informative equilibrium exists in unconstrained communication (i.e. CS). The condition (M) guarantees that \bar{b} is well-defined. In what follows, we will establish the existence of an equilibrium with two non-degenerate intervals at some $b \geq \bar{b}$.

Before we discuss the existence of an informative equilibrium when b is high, let us look at the issue of the truth-telling type. In communication with a single sender without overconfidence or garbling, CS have shown that the receiver's ideal action and that of the sender never coincide for $b > 0$ due to $U_{13} > 0$. This implies that no type reveals truthfully in equilibrium. However, as we will see shortly, if there are multiple senders to be treated equally, or if we introduce garbling or overconfidence, $U_{13} > 0$ no longer implies that a particular type of sender's ideal action (or distribution of the action) and that of the receiver never coincide. If they do coincide at some θ there may exist a T-equilibrium, where the truth-telling type reveals truthfully. In a T-equilibrium, as we have seen in the uniform-quadratic setting, we require an additional condition to the "no arbitrage" condition: any other types than the truth-telling type must not induce the action (or distribution) for the truth-telling type. This implies that there are an infinite number of intervals in the neighbourhood of the truth-telling type, or that the truth-telling type is internal to an interval and his desired distribution/action coincides with the one induced by his neighbouring types in the interval.

5.1 Anonymous Communication/Equal Treatment

Let us first consider direct communication with a single rational sender i , which is essentially equivalent to the general version of CS. Let $\bar{y}(\underline{a}, \bar{a})$ be the receiver's best response to the sender when his belief is such that $\theta \in [\underline{a}, \bar{a})$. CS have shown that communication an informative equilibrium cannot exist for $b > \bar{b}$, where the critical value \bar{b} is defined by

$$U(\bar{y}(0, 1), 0, \bar{b}) = U(\bar{y}(0, 0), 0, \bar{b}). \quad (18)$$

This condition implies that the sender with type $\theta = 0$ is indifferent between revealing truthfully and being completely disguised. From the assumptions on the utility functions it is easy to see that $\bar{y}(0, 1) > \bar{y}(0, 0)$ and thus $U(\bar{y}(0, 1), 0, b) > U(\bar{y}(0, 0), 0, b)$ for all $b > \bar{b}$.

In the following we will focus on the anonymous communication where $n = 2$, and show that when $b = \bar{b}$ there exists an informative equilibrium where the type space is partitioned by at least two non-degenerate intervals, and that by continuity there exists an informative equilibrium with $b \in [\bar{b}, \bar{b} + \epsilon]$ at least for some small ϵ . This result extends to the case where $n \geq 3$ but the exposition becomes much more complicated. As in the uniform-quadratic setting anything that enables the receiver to commit to equal treatment of all senders will have the same effect on communication as anonymity.

Let us denote the sender we focus on by i and the other sender by $-i$. The receiver's action conditional on two anonymous messages about sender types is given by

$$\begin{aligned} & \bar{y}(a_j, a_{j+1} \mid a_k, a_{k+1}) \\ &= \arg \max_y \frac{\int_{a_j}^{a_{j+1}} U(y, \theta_i, 0) f(\theta_i) d\theta_i}{\int_{a_j}^{a_{j+1}} f(\theta_i) d\theta_i} + \frac{\int_{a_k}^{a_{k+1}} U(y, \theta_{-i}, 0) f(\theta_{-i}) d\theta_{-i}}{\int_{a_k}^{a_{k+1}} f(\theta_{-i}) d\theta_{-i}}. \end{aligned} \quad (19)$$

That is, $\bar{y}(a_j, a_{j+1} \mid a_k, a_{k+1})$ is the receiver's best response when $\theta_i \in [a_j, a_{j+1})$ and $\theta_{-i} \in [a_k, a_{k+1})$. Let $\bar{y}_{Si}(\underline{a}, \bar{a})$ be the receiver's action from sender i 's viewpoint, conditional on the receiver's belief that $\theta_i \in [\underline{a}, \bar{a})$. Clearly $\bar{y}_{Si}(\underline{a}, \bar{a})$ is a random variable in any informative equilibrium. In the uniform-quadratic setting, in order to derive the sender's equilibrium strategy, it suffices to consider the expected value of the action, since sender i 's utility is affected only by the expected value and the variance of the receiver's action, and the variance is independent from sender i 's strategy. However, in the current setting we need to consider the entire distribution of the receiver's action because the utility is now affected by higher moments. In other words, we must fully take into account the fact that, from a sender's viewpoint, a message induces a particular *distribution* of the receiver's action.

The following lemma says that under anonymity, if $b = \bar{b}$ a sender with $\theta_i = 0$ strictly prefers revealing truthfully than being completely disguised.

Lemma 1 Suppose that the senders' partitional strategy is such that the type space is divided into $[0], (0, 1]$. In anonymous communication, if $b = \bar{b}$, sender i with $\theta_i = 0$ is strictly better off by inducing $\bar{y}_{Si}(0, 0)$ than $\bar{y}_{Si}(0, 1)$.

Proof. Suppose that sender i 's type is $\theta_i = 0$. If he follows this partitional strategy, he reveals truthfully, and symmetric strategies imply that if the other sender's type is $\theta_{-i} = 0$ he also reveals truthfully. However, the probability that $\theta_{-i} = 0$ is 0 since the sender's type is drawn from a continuous density function. Hence, almost surely, the receiver's action y^C is given by

$$y^C = \arg \max_y U(y, 0, 0) + \int_0^1 U(y, \theta_{-i}, 0) f(\theta_{-i}) d\theta_{-i}.$$

On the other hand, if the message of the sender i induces the belief $\theta_i \in (0, 1]$ the receiver's action is given by, almost surely,

$$y^N = \bar{y}(0, 1 \mid 0, 1) = \bar{y}(0, 1),$$

where $\bar{y}_i(0, 1)$ denotes the action in the uninformative equilibrium in CS. We have $\bar{y}(0, 1) \equiv y^N > y^C > \bar{y}(0, 0)$ since the receiver's utility function is supermodular in y and θ_i . Recall that by definition, at $b = \bar{b}$ the sender is indifferent between $\bar{y}(0, 1)$ and $\bar{y}(0, 0)$. By concavity of the utility function, the sender is strictly better off with y^C than y^N . Hence, the sender with $\theta_i = 0$ strictly prefers to reveal truthfully under the partitional strategy. ■

Lemma 1 says that the partitional strategy $[0], (0, 1]$ is (strictly) incentive compatible for $\theta_i = 0$. However, the above construction does not support a perfect Bayesian equilibrium, because a sender with $\theta_i = 0 + \epsilon$ for small ϵ will also prefer to induce $\bar{y}_{Si}(0, 0)$ rather than $\bar{y}_{Si}(0, 1)$. We use Lemma 1 in the following Proposition to show that there exists an equilibrium with two non-degenerate intervals $[0, a)$, $[a, 1]$.

Proposition 4 Suppose that $b = \bar{b}$. In anonymous communication, there exists a partially informative equilibrium where the type space is partitioned into two non-degenerate intervals.

Proof. In the following we will show that a symmetric sender strategy with two non-degenerate intervals supports a perfect Bayesian equilibrium. Suppose that the partitional strategy is such that $[0, a)$, and $[a, 1]$ for both senders. Suppose further that $\theta_i = a$. Define

$$\begin{aligned} V(0, a) &\equiv F(a)U(\bar{y}(0, a \mid 0, a), a, \bar{b}) + (1 - F(a))U(\bar{y}(0, a \mid a, 1), a, \bar{b}) \\ V(a, 1) &\equiv F(a)U(\bar{y}(a, 1 \mid 0, a), a, \bar{b}) + (1 - F(a))U(\bar{y}(a, 1 \mid a, 1), a, \bar{b}). \end{aligned}$$

$V(0, a)$ and $V(a, 1)$ are the expected utility of sender i with $\theta_i = a$ when his message induces the receiver's (random) action $\bar{y}_{Si}(0, a)$, and $\bar{y}_{Si}(a, 1)$, respectively. Define $A(a) \equiv V(0, a) - V(a, 1)$. The no arbitrage condition requires $A(a^*) = 0$. Clearly, $A(1) < 0$, since the receiver's action is, almost surely, either $\bar{y}(1, 1)$ or $\bar{y}(0, 1)$, and $\bar{y}(1, 1) > \bar{y}(0, 1)$ implies that the sender strictly prefers $\bar{y}(1, 1)$. On the other hand, from Lemma 1 we know that if $a = 0$, then $A(0) > 0$. Since $A(a)$ is a continuous function, by the intermediate value theorem there must be $a^* \in [0, 1]$ such that $A(a^*) = 0$. Moreover, $A(0) \neq 0$ and $A(1) \neq 0$. Therefore, there exists a $\theta_i = a^* \in (0, 1)$ such that the sender with $\theta_i = a^*$ is indifferent between inducing $\bar{y}_{Si}(0, a^*)$, and $\bar{y}_{Si}(a^*, 1)$.

It remains to show that the partitional strategy $[0, a^*), [a^*, 1]$ supports a perfect Bayesian equilibrium. Define

$$\begin{aligned} B(\theta_i) \equiv & [F(a^*)U(\bar{y}(0, a^* | 0, a^*), \theta_i, \bar{b}) + (1 - F(a^*))U(\bar{y}(0, a^* | a^*, 1), \theta_i, \bar{b})] \\ & - [F(a^*)U(\bar{y}(a^*, 1 | 0, a^*), \theta_i, \bar{b}) + (1 - F(a^*))U(\bar{y}(a^*, 1 | a^*, 1), \theta_i, \bar{b})]. \end{aligned}$$

$B(\theta_i)$ is an alternative representation of $A(a)$ as a function of θ_i given the partition $[0, a^*), [a^*, 1]$. In other words, $B(\theta_i)$ is the difference between the expected utilities of the sender with θ_i when his message induces $\bar{y}_{Si}(0, a^*)$ and $\bar{y}_{Si}(a^*, 1)$, respectively. Note that $\bar{y}(0, a^* | 0, a^*) < \bar{y}(a^*, 1 | 0, a^*)$, and $\bar{y}(0, a^* | a^*, 1) < \bar{y}(a^*, 1 | a^*, 1)$. Since $U_{12} > 0$, we have

$$\frac{dB(\theta_i)}{d\theta_i} < 0. \quad (20)$$

This serves as a sorting condition for every type of the sender. By definition $B(a^*) = 0$. Thus (20) implies that the sender with $\theta_i \leq a^*$ prefers to induce $\bar{y}_{Si}(0, a^*)$, while the sender with $\theta_i > a^*$ prefers to induce $\bar{y}_{Si}(a^*, 1)$. Hence, this partitional strategy is consistent with the receiver's belief. The receiver's action is given by (19), which is clearly a best response to the sender strategy described above. Hence, the partitional strategy with two non-degenerate intervals $[0, a^*), [a^*, 1]$ supports a perfect Bayesian equilibrium. ■

Corollary 1 By continuity, an informative equilibrium exists for $b \in [\bar{b}, \bar{b} + \epsilon]$ at least for some small ϵ .

The receiver strictly prefers the equilibrium with two intervals to the uninformative equilibrium, since conditional on any combination of messages in the informative equilibrium her expected utility is higher, as she can adjust her action accordingly. This, unlike the uniform-quadratic setting we have seen above, does not necessarily apply to the senders under this general setting. That is, although the expected utility the receiver obtains per worker $EU(y, \theta_i, 0)$ is higher in the informative equilibrium, this does not imply that $EU(y, \theta_i, b)$ is higher too.

5.2 Overconfidence/Garbling

We can show that the introduction of the garbling mechanism or an overconfident expert also extends the possibility of information transmission in this general setting with a single sender.⁴ Recall that in our model $1 - p$ denotes the probability that the overconfident sender observes pure noise, and $1 - q$ denotes the probability that the message is sent by the randomization device according to the ex ante distribution of the equilibrium message. In both cases, letting $\eta = p = q$ for convenience, the receiver's best response given a message indicating $\theta \in [a_j, a_{j+1})$ is

$$\bar{y}(a_j, a_{j+1}) = \arg \max_y \eta \frac{\int_{a_j}^{a_{j+1}} U(y, \theta, 0) f(\theta) d\theta}{\int_{a_j}^{a_{j+1}} f(\theta) d\theta} + (1 - \eta) \int_0^1 U(y, \theta, 0) f(\theta) d\theta. \quad (21)$$

The notion of weak response to a message is captured by the sensitivity parameter η : the message is weighted at $\eta < 1$ to take into account the possibility that it is uninformative.

Suppose that $b = \bar{b}$ and that the sender's type space is divided into $[0], (0, 1]$. In both the overconfidence and garbling cases, given that the message reports $\theta = 0$ and the receiver believes it, her best response is given by

$$y^C = \arg \max_y \eta U(y, 0, 0) + (1 - \eta) \int_0^1 U(y, \theta, 0) f(\theta) d\theta.$$

Clearly we have $\bar{y}(0, 1) \equiv y^N > y^C > \bar{y}(0, 0)$. Since the sender is indifferent between $\bar{y}(0, 1)$ and $\bar{y}(0, 0)$, by concavity of the utility function, the sender prefers y^C to y^N in the case of overconfidence (here the expected utility is calculated in terms of his wrong belief). Also when the garbling mechanism is used,

$$\eta U(y^C, 0, \bar{b}) + (1 - \eta) U(y(0, 1), 0, \bar{b}) > U(y(0, 0), 0, \bar{b}) = U(y(0, 1), 0, \bar{b}). \quad (22)$$

The left hand side of the inequality in (22) represents the expected utility of the sender with $\theta = 0$ when he tells the truth. The right hand side is his expected utility when he reports $\theta \in (0, 1]$. Hence, given the receiver's belief, the sender with $\theta = 0$ strictly prefers to reveal truthfully. Now a similar argument to the proof of Proposition 4 can be used to show that there exists a partition $[0, a^*], (a^*, 1]$ for some $a^* > 0$ that supports a perfect Bayesian equilibrium for any $0 < \eta < 1$. Moreover, by continuity the equilibrium with two intervals also exists for $b \in [\bar{b}, \bar{b} + \epsilon]$.

Clearly the receiver is better off in this equilibrium with two intervals than in the uninformative equilibrium, since the expected utility conditional on any message she has received is higher than the expected utility in the uninformative equilibrium. The same

⁴A similar result is independently obtained by Blume and Board (2006) in the context of garbling, but the garbling mechanism they adopt could not be reinterpreted as overconfidence.

reasoning does not apply to the sender in the general case, although both parties strictly prefer an informative equilibrium in the uniform-quadratic setting.

6 Concluding Remarks

This paper has provided a simple yet rich framework to study communication with anonymity, equal treatment, overconfident experts, garbling, and in public good provision. We have shown that, with all the communication features of concern in this paper, the receiver puts less weight on a message when making her decision compared with situations where a single rational sender communicates directly with the receiver. In the uniform-quadratic setting, we have characterized informative equilibria and demonstrated that weak response to a message may change the structure of informative equilibria dramatically. In particular, there exists no fully revealing equilibrium even in the absence of individual conflict of interest, but there may exist a type of sender whose ideal action coincides with that of the receiver (in expected terms) even in the presence of conflict. The most informative equilibrium may involve an infinite number of intervals, although it is not fully revealing. The structure of the most informative equilibrium is in stark contrast to that of CS, where there are a finite number of intervals in the type space when there is a conflict of interest between communicating parties.

We have argued that various constraints on communication such as anonymity, equal treatment, overconfidence, and garbling may improve information transmission when the level of conflict is large.

To our knowledge, the framework we have developed above offers the first attempt at the formal analysis of anonymity and overconfidence in communication. Anonymity has been an important concept in the social choice literature, but typically it refers to a certain property in a social choice function rather than a characteristic of messages the decision maker uses in choosing her action. By introducing anonymity into the standard cheap talk model, we are able to illustrate the role of anonymity in the receiver's decision making and its consequences on information revelation. While the economic literature on overconfidence has been concerned mainly with its implication for markets, our focus is on the credibility of advice from an overconfident expert.

Our paper also sheds new light on communication in public good provision. Much of the literature has focused on situations in which each agent determines his own contribution under a certain provision rule. In contrast, our model is particularly relevant to communication in public good provision where a decision maker, who cannot *ex ante* commit to a provision rule, communicates with the members of a group and chooses the quality or quantity of a public good at the same or no cost (contribution) for each of them. We

have demonstrated that, when there is a personal bliss point (ideal action) in the agent's preference, exaggeration will be an important issue.

It has already been known since Myerson (1986) and Forges (1986) that garbling can improve welfare, and more recently Krishna and Morgan (2004), Blume and Board (2006) and others have proposed garbling devices to achieve Pareto improvement. This paper contributes to the literature too, by providing a systematic analysis of a well-defined garbling mechanism and by illustrating how it potentially changes the structure of informative equilibria, even with a very small possibility of garbling.

7 Appendix I: Rational Sender with $p < 1$

Let $E_s[\theta \mid m]$ be the expected value of θ conditional on the message, given that the sender observes the true θ . As in the main text the receiver's best response is given by

$$y_R(m) = pE_s[\theta \mid m] + (1 - p)\frac{1}{2}.$$

On the other hand the sender's ideal action given that he has observed θ is, as he recognizes p correctly,

$$y^S(\theta) = p\theta + (1 - p)\frac{1}{2} + b. \quad (23)$$

Note that since

$$y^R(\theta) = p\theta + (1 - p)\frac{1}{2},$$

both parties' ideal actions for θ never coincide if $b > 0$. This implies that, like CS, there does not exist a truth-telling type and the sender has incentives to overstate only.

Let us find the equilibrium partitions. Since the utility function is symmetric with respect to y , for (A) to be true, it must be that the action that the sender with observed type a_j wishes to induce, $pa_j + (1 - p)\frac{1}{2} + b$, lies exactly halfway in between these two actions induced by two different messages

$$\begin{aligned} pa_j + (1 - p)\frac{1}{2} + b &= \frac{\bar{y}(a_{j-1}, a_j) + \bar{y}(a_j, a_{j+1})}{2} \\ &= p \left[\frac{a_{j-1} + a_j}{4} + \frac{a_j + a_{j+1}}{4} \right] + (1 - p)\frac{1}{2}. \end{aligned} \quad (24)$$

From (24) we obtain a second-order difference equation

$$a_{j+1} - 2a_j + a_{j-1} = \frac{4b}{p}.$$

Solving for $a_0 = 0$ and $a_2 = 1$ we obtain

$$a_1 = \frac{1}{2} - \frac{2b}{p},$$

which gives the equilibrium partition with two intervals: $a_0 = 0, a_1 = \frac{1}{2} - \frac{2b}{p}, a_2 = 1$. Thus this equilibrium can be supported if $a_1 \geq 0$ or

$$b \leq \frac{p}{4} = \bar{b}.$$

Hence, the largest bias that allows information transmission to occur is decreasing in p . In contrast, if the sender is overconfident \bar{b} is increasing in p , as we have seen in (17).

8 Appendix II: Proof of Proposition 3

8.1 Preliminaries

Before we prove the Proposition, we provide some useful lemmas and discuss how we construct the main proof. Let us call a sequence $\{a_0, a_1, \dots, a_J\}$ that satisfies (A) a "*solution*" to (A). The monotonicity condition (M) in CS requires that, for given γ and b , if we have two solutions a^+ and a^{++} with $a_0^+ = a_0^{++}$ and $a_1^+ > a_1^{++}$, then $a_j^+ > a_j^{++}$ for all $j = 2, 3, \dots$. In other words, (M) says that starting from a_0 , all solutions to (A) must move up or down together.

Lemma 2 Any solutions to (A) satisfies the monotonicity condition (M) in CS

Proof. Explicitly solving (12), which is replicated below

$$\gamma a_{j+1} - (4 - 2\gamma)a_j + \gamma a_{j-1} = 4b + 2\gamma - 2, \quad (25)$$

we obtain

$$a_j = \frac{\gamma a_1 - \hat{\theta}(2 - 2\gamma - 2\sqrt{1 - \gamma})}{4\sqrt{1 - \gamma}} \left(\frac{2 - \gamma + 2\sqrt{1 - \gamma}}{\gamma} \right)^j + \frac{-\gamma a_1 - \hat{\theta}(2 - 2\gamma + 2\sqrt{1 - \gamma})}{4\sqrt{1 - \gamma}} \left(\frac{2 - \gamma - 2\sqrt{1 - \gamma}}{\gamma} \right)^j + \hat{\theta},$$

where $\hat{\theta} \equiv \frac{1}{2} - \frac{b}{1 - \gamma}$. Since we have

$$\frac{da_j}{da_1} = \frac{\gamma}{4\sqrt{1 - \gamma}} \left[\left(\frac{2 - \gamma + 2\sqrt{1 - \gamma}}{\gamma} \right)^j - \left(\frac{2 - \gamma - 2\sqrt{1 - \gamma}}{\gamma} \right)^j \right] > 0,$$

all solutions to (A) must move up or down together. This is the definition of condition (M) in CS. ■

Since our uniform-quadratic setting with the sensitivity parameter γ satisfies (M), we can resort to various useful results provided by CS. In particular, the equilibrium partition with size J is unique.

In order to show that the players' expected utility is higher in an equilibrium with more intervals, CS deform the partition with J intervals to that with $J + 1$ intervals, by continuously increasing the player's expected utility throughout the deformation. We follow this method, but we need to proceed by two step deformation, rather than one.

Let $a(J)$ be the equilibrium partition of size J . We show that $a(J)$ can be deformed to $a(J + 1)$ by two steps, continuously increasing the players' expected utility in each step. Here we consider the case where the truth-telling type exists, or $\hat{\theta} \in [0, 1]$. The case where $\hat{\theta} \notin [0, 1]$ can be proven similarly, by using the first deformation only.

Let the sub-partition of $a(J)$ equal or below $\hat{\theta}$ be $\underline{a}(J) \equiv (a_0(J), a_2(J), \dots, a_K(J))$ where $a_0(J) = 0$. Also, suppose that $a_K(J)$ is closer to $\hat{\theta}$ than $a_{K+1}(J)$ is, in other words, $\hat{\theta} - a_K(J) < a_{K+1}(J) - \hat{\theta}$. In the following we proceed in two steps:

1. We fix $a_K(J)$ and make the sub-partition $(a_K(J), a_{K+1}(J), \dots, a_J(J))$ deform continuously to $(a_K(J), a_{K+1}(J+1), a_{K+2}(J+1), \dots, a_{J+1}(J+1))$, increasing the expected utility.
 2. We make the sub-partition $(a_0(J), a_1(J), \dots, a_K(J))$ deform continuously to $(a_0(J+1), a_1(J+1), \dots, a_K(J+1))$, increasing the expected utility.
- If $\hat{\theta} - a_K(J) \geq a_{K+1}(J) - \hat{\theta}$ then the first step deforms $(a_0(J), a_1(J), \dots, a_K(J), a_{K+1}(J))$ to $(a_0(J+1), a_1(J+1), \dots, a_{K+1}(J+1), a_{K+1}(J))$ while fixing $a_{K+1}(J)$, and the second step deforms $(a_{K+1}(J), a_{K+2}(J), \dots, a_J(J))$ to $(a_{K+2}(J+1), a_{K+3}(J+1), \dots, a_{J+1}(J+1))$. Except for this the same method and result as the case where $\hat{\theta} - a_K(J) < a_{K+1}(J) - \hat{\theta}$ apply.

Lemma 3 If $a(J)$ and $a(J + 1)$ are two equilibrium partitions for the same values of b and γ , then $a_{j-1}(J) < a_j(J + 1) < a_j(J)$.

Proof. See Lemma 3 (p.1446) in CS. The proof follows directly from (M). ■

The first step of deformation is carried out as follows. Let $(a_K^x, a_{K+1}^x, \dots, a_j^x, \dots, a_{J+1}^x)$ be the sub-partition that satisfies (A) for all $j = K + 1, K + 2, \dots, J$ with $a_K^x = a_K(J)$, $a_j^x = x$ and $a_{J+1}^x = 1$. If $x = a_{J-1}(J)$ then $a_{K+1}^x = a_K^x = a_K(J)$. If $x = a_J(J + 1)$ then we have $(a_K(J), a_{K+1}(J + 1), \dots, a_J(J + 1))$, where (A) is satisfied for all $j = K + 2, K + 3, \dots, J$. We are going to show that, if $x \in [a_{J-1}(J), a_J(J + 1)]$, which is again a non-degenerate interval by Lemma 3, then the sender's expected utility is strictly increasing in x .

In the second step, let $(a_0^z, a_1^z, \dots, a_j^z, \dots, a_K^z)$ be the sub-partition that satisfies (A) for $j = 1, 2, \dots, K - 1$, with $a_0^z = 0$ and $a_K^z = z$. If $z = a_K(J)$ then $a_j^z = a_j(J)$ for all $j = 0, 1, \dots, K$. If $z = a_K(J + 1)$ then $a_j^z = a_j(J + 1)$ for all $j = 0, 1, \dots, K$. We will show that when $z \in [a_K(J + 1), a_K(J)]$, which is again a non-degenerate interval by Lemma 3, the sender's expected utility is strictly decreasing in z .

Lemma 4 Suppose that $\{a_0, a_1, \dots, a_j, \dots, a_J\}$ is a solution to (A). Then for all $j = 1, 2, \dots, J-1$ if $a_j > (<) \hat{\theta}$ then $a_j - a_{j-1} < a_{j+1} - a_j$ ($a_j - a_{j-1} > a_{j+1} - a_j$). If $a_j = \hat{\theta}$ then $a_j - a_{j-1} = a_{j+1} - a_j$.

Proof. The sequences that satisfy (A) are described by (12). Rearranging (12) we have

$$(a_{j+1} - a_j) - (a_j - a_{j-1}) = \frac{4a_j + 4b + 2\gamma - 2}{\gamma} - 4a_j. \quad (26)$$

The left hand side $(a_{j+1} - a_j) - (a_j - a_{j-1}) = 0$ if

$$\begin{aligned} \frac{4a_j + 4b + 2\gamma - 2}{\gamma} - 4a_j &= 0 \Rightarrow \\ 4a_j(1 - \gamma) &= -4b - 2\gamma + 2 \Rightarrow \\ a_j &= \frac{1}{2} - \frac{b}{1 - \gamma} \equiv \hat{\theta}. \end{aligned}$$

Since the right hand side of (26) is increasing in a_j , if $a_j > \hat{\theta}$ then $(a_{j+1} - a_j) - (a_j - a_{j-1}) > 0$, and if $a_j < \hat{\theta}$ then $(a_{j+1} - a_j) - (a_j - a_{j-1}) < 0$. ■

The above lemma says that an interval $[a_{j+1}, a_j]$ is longer (shorter) than the previous interval $[a_{j-1}, a_j]$ when $a_j > (<) \hat{\theta}$. The intuition is captured in Figure 2. The following Lemma is similar but cannot be implied by Lemma 4. Since by definition a_K^x and a_{K+1}^z are fixed throughout the respective deformation, (A) is not satisfied at $a_j = a_{K+1}^x$ or $a_j = a_K^z$.

Lemma 5 $a_{K+1}^x - a_K^x < a_{K+2}^x - a_{K+1}^x$ and $a_K^z - a_{K-1}^z > a_{K+1}^z - a_K^z$.

Proof. From Lemma 4 we have $a_{K+1}^x - \tilde{a}_K < a_{K+2}^x - a_{K+1}^x$ where \tilde{a}_K is defined such that $(a_{j-1} = \tilde{a}_K, a_j = a_{K+1}^x, a_{j+1} = a_{K+2}^x)$ satisfies (12). Since $a_K(J+1) < \tilde{a}_K < a_K(J) = a_K^x$ from Lemma 3, we have $a_{K+1}^x - a_K^x < a_{K+2}^x - a_{K+1}^x$. This proves the first part of the Lemma.

Similarly we have $a_K^z - \check{a}_{K+1} \geq a_{K+1}^z - a_K^z$ where \check{a}_{K+1} is defined such that $(a_{j-1} = a_{K-1}^z, a_j = a_K^z, a_{j+1} = \check{a}_{K+1})$ satisfies (12). Lemma 3 implies $a_{K+1}^z = a_{K+1}(J+1) < \check{a}_{K+1} < a_{K+1}(J)$. Hence we have $a_K^z - a_{K-1}^z > a_{K+1}^z - a_K^z$. ■

8.2 Anonymous Communication/Equal Treatment

8.2.1 Sender

The receiver's action from a sender's viewpoint is a random variable, and since the utility functions are quadratic, we can separate the expected value terms and the variance terms. The sender's utility in this separated form conditional of his report is given by

$$\begin{aligned} E \left[-(y_i(m_i) - (\theta_i + b))^2 \right] \\ = -\text{var}(y_i(m_i)) - (Ey_i(m_i))^2 + 2(\theta_i + b)Ey_i(m_i) - (\theta_i + b)^2 \\ = -\text{var}(y_i) - (Ey_i(m_i) - (\theta_i + b))^2, \end{aligned} \quad (27)$$

where

$$Ey_i(m_i) = E[\theta_i | m_i] + \frac{n-1}{2n}.$$

The variance term is independent of the sender's message since the randomness is caused by the other senders' messages unobservable to the sender. Let the sender i 's expected type given his message be \hat{a}_i . Since the receiver's action is the mean of all posterior expected types, from sender i 's viewpoint

$$\text{var}(y_i) = \text{var}\left(\frac{1}{n}\left(\sum_{l \neq i} \hat{a}_l + \hat{a}_i\right)\right) = \frac{1}{n^2} \text{var}\left(\sum_{l \neq i} \hat{a}_l + \hat{a}_i\right) = \frac{n-1}{n^2} \text{var}(\hat{a}),$$

where $\text{var}(\hat{a})$ is the variance of the expected type of a sender given his equilibrium strategy. The last equality follows from independent type distributions and symmetric sender strategies.

The expected utility for the first part of deformation is given by

$$\begin{aligned} EU^S &\equiv - \sum_{j=1}^K \int_{a_{j-1}^x}^{a_j^x} \left(\frac{a_{j-1} + a_j}{2n} + \frac{n-1}{2n} - b - \theta \right)^2 d\theta \\ &- \sum_{j=K+1}^{J+1} \int_{a_{j-1}^x}^{a_j^x} \left(\frac{a_{j-1}^x + a_j^x}{2n} + \frac{n-1}{2n} - b - \theta \right)^2 d\theta \\ &- \frac{n-1}{n^2} \sum_{j=1}^K \left[\frac{(a_j - a_{j-1})(a_{j-1} + a_j)^2}{4} - \frac{1}{4} \right] \\ &- \frac{n-1}{n^2} \sum_{j=K+1}^{J+1} \left[\frac{(a_j^x - a_{j-1}^x)(a_{j-1}^x + a_j^x)^2}{4} - \frac{1}{4} \right]. \end{aligned}$$

It follows that

$$\begin{aligned} \frac{dEU^S}{dx} &\equiv \sum_{j=K+1}^{J+1} \frac{da_j^x}{dx} \left\{ - \left(\frac{a_{j-1}^x + a_j^x}{2n} + \frac{n-1}{2n} - b - a_j^x \right)^2 + \left(\frac{a_j^x + a_{j+1}^x}{2n} + \frac{n-1}{2n} - b - a_j^x \right)^2 \right. \\ &- \frac{1}{n} \left[\int_{a_{j-1}^x}^{a_j^x} \left(\frac{a_{j-1}^x + a_j^x}{2n} + \frac{n-1}{2n} - b - \theta \right) d\theta + \int_{a_j^x}^{a_{j+1}^x} \left(\frac{a_j^x + a_{j+1}^x}{2n} + \frac{n-1}{2n} - b - \theta \right) d\theta \right] \\ &\left. - \left[\frac{n-1}{2n^2} (a_{j+1}^x)^2 - (a_{j-1}^x)^2 + \frac{(a_{j-1}^x + a_j^x)^2 - (a_j^x + a_{j+1}^x)^2}{2} \right] \right\}. \end{aligned}$$

We have⁵

$$\begin{aligned}
& - \left(\frac{a_{j-1}^x + a_j^x}{2n} + \frac{n-1}{2n} - b - a_j^x \right)^2 + \left(\frac{a_j^x + a_{j+1}^x}{2n} + \frac{n-1}{2n} - b - a_j^x \right)^2 \\
& = \frac{n-1}{2n^2} (a_{j+1}^x - a_{j-1}^x)(1 - 2a_j^x) - \frac{b(a_{j+1}^x - a_{j-1}^x)}{n} + \frac{(a_{j+1}^x - a_{j-1}^x)(a_{j-1}^x - 2a_j^x + a_{j+1}^x)}{4n^2}.
\end{aligned}$$

Also the second line,

$$\begin{aligned}
& - \frac{1}{n} \left[\int_{a_{j-1}^x}^{a_j^x} \left(\frac{a_{j-1}^x + a_j^x}{2n} + \frac{n-1}{2n} - b - \theta \right) d\theta + \int_{a_{j-1}^x}^{a_j^x} \left(\frac{a_j^x + a_{j+1}^x}{2n} + \frac{n-1}{2n} - b - \theta \right) d\theta \right] \\
& = \frac{n-1}{2n^2} [(a_{j+1}^x)^2 - (a_{j-1}^x)^2 - (a_{j+1}^x - a_{j-1}^x)] + \frac{b(a_{j+1}^x - a_{j-1}^x)}{n}.
\end{aligned}$$

Hence,

$$\begin{aligned}
& - \left(\frac{a_{j-1}^x + a_j^x}{2n} + \frac{n-1}{2n} - b - a_j^x \right)^2 + \left(\frac{a_j^x + a_{j+1}^x}{2n} + \frac{n-1}{2n} - b - a_j^x \right)^2 \\
& - \frac{1}{n} \left[\int_{a_{j-1}^x}^{a_j^x} \left(\frac{a_{j-1}^x + a_j^x}{2n} + \frac{n-1}{2n} - b - \theta \right) d\theta + \int_{a_j^x}^{a_{j+1}^x} \left(\frac{a_j^x + a_{j+1}^x}{2n} + \frac{n-1}{2n} - b - \theta \right) d\theta \right] \\
& - \frac{n-1}{2n^2} \left[(a_{j+1}^x)^2 - (a_{j-1}^x)^2 + \frac{(a_{j-1}^x + a_j^x)^2 - (a_j^x + a_{j+1}^x)^2}{2} \right] \\
& = \frac{a_{j+1}^x - a_{j-1}^x}{2n} \left[\frac{a_{j-1}^x - 2a_j^x + a_{j+1}^x}{2} \right] > 0.
\end{aligned}$$

The inequality follows because from Lemmas 4 and 5, we have $a_j - a_{j-1} < a_{j+1} - a_j \Rightarrow a_{j-1}^x - 2a_j^x + a_{j+1}^x > 0$ for all $j = K+1, K+2, \dots, J$. We have $\frac{da_j^x}{dx} > 0$ by (M). It follows that

$$\frac{dEUS}{dx} \equiv \sum_{j=K+1}^{J+1} \frac{da_j^x}{dx} \left\{ \frac{a_{j+1}^x - a_{j-1}^x}{2n} \left[\frac{a_{j-1}^x - 2a_j^x + a_{j+1}^x}{2} \right] \right\} > 0.$$

⁵For $j = K+2, K+3, \dots, J-1$ we can use the fact that a_j^x satisfies (A) or

$$- \left(\frac{a_{j-1}^x + a_j^x}{2n} + \frac{n-1}{2n} - b - a_j^x \right)^2 + \left(\frac{a_j^x + a_{j+1}^x}{2n} + \frac{n-1}{2n} - b - a_j^x \right)^2 = 0$$

to simplify the calculation, although later exposition will become more complex because this does not apply to $j = K$.

Let us look at the second part of deformation

$$\begin{aligned}
\frac{dEU^S}{dz} &\equiv \sum_{j=1}^K \frac{da_j^z}{dz} \left\{ - \left(\frac{a_{j-1}^z + a_j^z}{2n} + \frac{n-1}{2n} - b - a_j^z \right)^2 + \left(\frac{a_j^z + a_{j+1}^z}{2n} + \frac{n-1}{2n} - b - a_j^z \right)^2 \right. \\
&\quad \left. - \frac{1}{n} \left[\int_{a_{j-1}^z}^{a_j^z} \left(\frac{a_{j-1}^z + a_j^z}{2n} + \frac{n-1}{2n} - b - \theta \right) d\theta + \int_{a_j^z}^{a_{j+1}^z} \left(\frac{a_j^z + a_{j+1}^z}{2n} + \frac{n-1}{2n} - b - \theta \right) d\theta \right] \right. \\
&\quad \left. - \frac{n-1}{2n^2} \left[(a_{j+1}^z)^2 - (a_{j-1}^z)^2 - \frac{(a_{j-1}^z + a_j^z)^2 - (a_j^z + a_{j+1}^z)^2}{2} \right] \right\} \\
&= \sum_{j=1}^K \frac{da_j^z}{dz} \left\{ \frac{a_{j+1}^z - a_{j-1}^z}{2n} \left[\frac{a_{j-1}^z - 2a_j^z + a_{j+1}^z}{2} \right] \right\} < 0.
\end{aligned}$$

The inequality follows because $\frac{da_j^z}{dz} > 0$ by (M), and from $a_0, a_1, \dots, a_K \leq \hat{\theta}$ and Lemmas 4 and 5 we have $a_j - a_{j-1} > a_{j+1} - a_j \Rightarrow a_{j-1}^z - 2a_j^z + a_{j+1}^z < 0$ for all $j = 1, 2, \dots, K$.

Since we have completed the deformation from $a(J)$ to $a(J+1)$ by two steps while increasing the expected utility, we conclude that the sender's expected utility is higher in an equilibrium with more intervals.

8.2.2 Receiver

Since the receiver's utility is the sum of the senders' utilities without bias ($b = 0$), we can apply the above result for a sender's expected utility directly to show that the receiver's expected utility is higher with an equilibrium with more intervals.

8.3 Garbling

8.3.1 Sender

The sender's expected utility for the first part of deformation is given by

$$\begin{aligned}
EU^S &\equiv -q \left[\sum_{j=1}^K \int_{a_{j-1}}^{a_j} \left(q \frac{a_{j-1} + a_j}{2} + \frac{1-q}{2} - b - \theta \right)^2 d\theta \right. \\
&\quad \left. + \sum_{j=K+1}^{J+1} \int_{a_{j-1}^x}^{a_j^x} \left(q \frac{a_{j-1}^x + a_j^x}{2} + \frac{1-q}{2} - b - \theta \right)^2 d\theta \right] \\
&\quad - (1-q) \left[\sum_{j=1}^K (a_j - a_{j-1}) \int_0^1 \left(q \frac{a_{j-1} + a_j}{2} + \frac{1-q}{2} - b - \theta \right)^2 d\theta \right. \\
&\quad \left. + \sum_{j=K+1}^{J+1} (a_j^x - a_{j-1}^x) \int_0^1 \left(q \frac{a_{j-1}^x + a_j^x}{2} + \frac{1-q}{2} - b - \theta \right)^2 d\theta \right].
\end{aligned}$$

It follows that

$$\begin{aligned}
\frac{dEU^S}{dx} &\equiv \sum_{j=K+1}^{J+1} \frac{da_j^x}{dx} \times \\
&\left\{ -q \left[\left(q \frac{a_{j-1}^x + a_j^x}{2} + \frac{1-q}{2} - b - a_j^x \right)^2 - \left(q \frac{a_j^x + a_{j+1}^x}{2} + \frac{1-q}{2} - b - a_j^x \right)^2 \right. \right. \\
&\quad \left. \left. + \int_{a_{j-1}^x}^{a_j^x} \left(q \frac{a_{j-1}^x + a_j^x}{2} + \frac{1-q}{2} - b - \theta \right) d\theta + \int_{a_j^x}^{a_{j+1}^x} \left(q \frac{a_j^x + a_{j+1}^x}{2} + \frac{1-q}{2} - b - \theta \right) d\theta \right] \right. \\
&\quad \left. - (1-q) \left[\int_0^1 \left(q \frac{a_{j-1}^x + a_j^x}{2} + \frac{1-q}{2} - b - \theta \right)^2 d\theta - \int_0^1 \left(q \frac{a_j^x + a_{j+1}^x}{2} + \frac{1-q}{2} - b - \theta \right)^2 d\theta \right. \right. \\
&\quad \left. \left. + q(a_j^x - a_{j-1}^x) \int_0^1 \left(q \frac{a_{j-1}^x + a_j^x}{2} + \frac{1-q}{2} - b - \theta \right) d\theta \right. \right. \\
&\quad \left. \left. + q(a_{j+1}^x - a_j^x) \int_0^1 \left(q \frac{a_j^x + a_{j+1}^x}{2} + \frac{1-q}{2} - b - \theta \right) d\theta \right] \right\} \\
&= \sum_{j=K+1}^{J+1} \frac{da_j^x}{dx} \frac{q(a_{j+1}^x - a_{j-1}^x)}{4} [4b(1-q) + (2-3q+2q^2)(a_{j-1}^x + a_{j+1}^x - 1) - q(2a_j^x - 1)].
\end{aligned}$$

Since $\frac{da_j^x}{dx} > 0$, the sign of $\frac{dEU^S}{dx}$ depends on the sign of the terms in the square brackets.

For all $j = K, K+1, \dots, J$,

$$\begin{aligned}
&4b(1-q) + (2-3q+2q^2)(a_{j-1}^x + a_{j+1}^x - 1) - q(2a_j^x - 1) \\
&> 4b(1-q) + (2-4q+2q^2)(a_{j-1}^x + a_{j+1}^x - 1) \\
&= 4b(1-q) + 2(1-q)^2(a_{j-1}^x + a_{j+1}^x - 1) \\
&> 4b(1-q) + 2(1-q)^2 \left(\frac{1}{2} - \frac{b}{1-q} + \frac{1}{2} - \frac{b}{1-q} - 1 \right) \\
&= 4b(1-q) - 4b(1-q) = 0.
\end{aligned} \tag{28}$$

The first inequality follows since from Lemmas 4 and 5 we have $a_j - a_{j-1} < a_{j+1} - a_j \Rightarrow a_{j-1}^x + a_{j+1}^x - 1 > 2a_j^x - 1$. The second inequality follows from $a_{K+2}^x, \dots, a_{J+1}^x > \hat{\theta} \equiv 1/2 - \frac{b}{1-q}$ and from the assumption that $\hat{\theta} - a_K(J) \leq a_{K+1}(J) - \hat{\theta}$, which implies $a_K^x + a_{K+2}^x \geq 2\hat{\theta}$ where $a_K^x = a_K(J)$ and $a_{K+2}^x \geq a_{K+1}(J)$. (Note that $a_{K+2}^x = a_{K+1}(J)$ when $x = a_{J-1}(J)$.) Hence we have $\frac{dEU^S}{dx} > 0$, so EU^S is increasing throughout the first part of deformation.

Let us look at the second part of deformation.

$$\begin{aligned}
\frac{dEU^S}{dz} &\equiv \sum_{j=1}^K \frac{da_j^z}{dz} \times \\
&\left\{ -q \left[\left(q \frac{a_{j-1}^z + a_j^z}{2} + \frac{1-q}{2} - b - a_j^z \right)^2 - \left(q \frac{a_j^z + a_{j+1}^z}{2} + \frac{1-q}{2} - b - a_j^z \right)^2 \right. \right. \\
&\quad \left. \left. + \int_{a_{j-1}^z}^{a_j^z} \left(q \frac{a_{j-1}^z + a_j^z}{2} + \frac{1-q}{2} - b - \theta \right) d\theta + \int_{a_j^z}^{a_{j+1}^z} \left(q \frac{a_j^z + a_{j+1}^z}{2} + \frac{1-q}{2} - b - \theta \right) d\theta \right] \right. \\
&\quad \left. - (1-q) \left[\int_0^1 \left(q \frac{a_{j-1}^z + a_j^z}{2} + \frac{1-q}{2} - b - \theta \right)^2 d\theta - \int_0^1 \left(q \frac{a_j^z + a_{j+1}^z}{2} + \frac{1-q}{2} - b - \theta \right)^2 d\theta \right] \right. \\
&\quad \left. + q(a_j^z - a_{j-1}^z) \int_0^1 \left(q \frac{a_{j-1}^z + a_j^z}{2} + \frac{1-q}{2} - b - \theta \right) d\theta \right. \\
&\quad \left. + q(a_{j+1}^z - a_j^z) \int_0^1 \left(q \frac{a_j^z + a_{j+1}^z}{2} + \frac{1-q}{2} - b - \theta \right) d\theta \right\} \\
&= \sum_{j=1}^K \frac{da_j^z}{dz} \frac{q(a_{j+1}^z - a_{j-1}^z)}{4} [4b(1-q) + (2-3q+2q^2)(a_{j-1}^z + a_{j+1}^z - 1) - q(2a_j^z - 1)].
\end{aligned}$$

We have $\frac{da_j^z}{dz} > 0$ from (M). The terms in the square brackets are negative since, for all $j = 1, 2, \dots, K$.

$$\begin{aligned}
&4b(1-q) + (2-3q+2q^2)(a_{j-1}^z + a_{j+1}^z - 1) - q(2a_j^z - 1) \\
&< 4b(1-q) + (2-4q+2q^2)(2a_j^z - 1) \\
&= 4b(1-q) + 2(1-q)^2(2a_j^z - 1) \\
&< 4b(1-q) + 2(1-q)^2 \left(\frac{1}{2} - \frac{b}{1-q} + \frac{1}{2} - \frac{b}{1-q} - 1 \right) \\
&= 4b(1-q) - 4(1-q)b = 0.
\end{aligned} \tag{29}$$

The first inequality follows because Lemmas 4 and 5 imply $a_j - a_{j-1} > a_{j+1} - a_j \Rightarrow a_{j-1}^x + a_{j+1}^x - 1 < 2a_j^x - 1$. The second inequality follows from $a_0^z, a_1^z, \dots, a_K^z < \hat{\theta} \equiv 1/2 - \frac{b}{1-q}$. Therefore, EU^S is increasing throughout the second part of deformation for which z decreases from $a_K(J)$ to $a_K(J+1)$.

Since we have completed the deformation from $a(J)$ to $a(J+1)$ by two steps while increasing the expected utility, we conclude that the sender's expected utility is higher in an equilibrium with more intervals.

8.3.2 Receiver

Following the above two-step deformation, the receiver's expected utility for the first part of deformation is given by

$$\begin{aligned}
EU^R \equiv & -q \left[\sum_{j=1}^K \int_{a'_{j-1}}^{a'_j} \left(q \frac{a_{j-1} + a_j}{2} + \frac{1-q}{2} - \theta \right)^2 d\theta \right. \\
& + \left. \sum_{j=K+1}^{J+1} \int_{a^x_{j-1}}^{a^x_j} \left(q \frac{a^x_{j-1} + a^x_j}{2} + \frac{1-q}{2} - \theta \right)^2 d\theta \right] \\
& - (1-q) \left[\sum_{j=1}^K (a_j - a_{j-1}) \int_0^1 \left(q \frac{a_{j-1} + a_j}{2} + \frac{1-q}{2} - \theta \right)^2 d\theta \right. \\
& + \left. \sum_{j=K+1}^{J+1} (a^x_j - a^x_{j-1}) \int_0^1 \left(q \frac{a^x_{j-1} + a^x_j}{2} + \frac{1-q}{2} - \theta \right)^2 d\theta \right].
\end{aligned}$$

Note that the expected utility is identical to that of the sender, except that $b = 0$ for the receiver. Therefore, in order to show that the receiver's expected utility is higher in an equilibrium with more intervals, we can directly apply the the argument we have used for the sender's expected utility.

8.4 Overconfident Expert

8.4.1 Sender

If we consider that the expected utility is calculated according to his (wrong) belief that his observation of his type is always correct, his expected utility is given by

$$\begin{aligned}
EU^S \equiv & - \sum_{j=1}^K \int_{a_{j-1}}^{a_j} \left(p \frac{a_{j-1} + a_j}{2} + \frac{1-p}{2} - b - \theta \right)^2 d\theta \\
& - \sum_{j=K+1}^{J+1} \int_{a_{j-1}}^{a_j} \left(p \frac{a_{j-1} + a_j}{2} + \frac{1-p}{2} - b - \theta \right)^2 d\theta.
\end{aligned}$$

Note that this expression is the same as the sender's expected utility with garbling, except that the variance term is absent and that the weight on the message is given here by p rather than q . Therefore, we can directly apply the proof for the garbling case (without the variance term) and prove that the expected utility is strictly increasing in the number of the intervals.

If we assume that the sender's expected utility is the expected value of the actual realization of payoff, the expected utility is identical to that with garbling except that the sensitivity parameter is p rather than q . Again, we can directly apply the proof for the garbling case.

8.4.2 Receiver

The receiver's expected utility is identical to that with garbling except that the sensitivity parameter is p rather than q . We can directly apply the proof for the garbling case.

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